When to Drill?

Trigger Prices for the Arctic National Wildlife Refuge

by

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ABSTRACT

Exploring for the oil that may lie beneath the Arctic National Wildlife Refuge (ANWR) has sparked a heated debate between those wishing to preserve pristine arctic wilderness and those seeking to reduce the reliance of the United States on imported, crude oil. It is clear that an undisturbed ANWR is important (of value) to many individuals. It is also clear that the development and delivery of any oil beneath the ANWR will be a costly and risky proposition. This paper views the decision within a *real-option framework*, where the ANWR is currently providing an amenity dividend, where there are significant costs to field development, production, and delivery to west-coast refineries, and where the price of oil may be evolving according to geometric Brownian motion (GBM) or a mean-reverting (M-R) process. For GBM we derive an analytical expression for the trigger-price for crude oil, the price which would make field development optimal. This trigger price depends on eight parameters describing field development and production, the mean drift in the price of oil and its standard deviation, and the size of the amenity dividend which would be lost if field development takes place. We vary the amenity dividend from \$200 million to \$300 million per year and compute the price which would trigger field development. For M-R oil prices, one cannot obtain an analytic expression for the trigger price, but we can numerically solve for the value function for the ANWR and identify the price trigger from the value function. We then compare the price triggers for GBM and the M-R process.

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I. Introduction and Overview

On Wednesday, March 19th, 2003, by a vote of 52-48, the U.S. Senate, removed from a budget resolution a White House-backed provision that would have allowed the U.S. Congress to lift a ban on drilling for oil beneath the coastal plain of the Arctic National Wildlife Refuge (ANWR). Conservationists viewed the vote as a victory, and political analysts believe it is unlikely that the Bush Administration will re-introduce any similar provisions in 2003 or 2004. If, however, George W. Bush is re-elected in 2004, and if the Republicans increase their majority in the U.S. Senate, it is likely that a bill or provision to lift the ban would be re-introduced in 2005.

Given the possibility of a two-year hiatus on the ANWR debate, it might be appropriate to review the issues and to re-examine the question from a different perspective. The perspective we propose is the real-option framework described in Dixit and Pindyck (1994) and Trigeorgis (1996). This framework is especially well-suited to evaluate investments which are risky and impossible, or very costly, to reverse. If an investment can be viewed as a binary (0-1) variable, then the real-option approach becomes a "stopping-rule problem." In a stopping-rule problem one is continuously trying to determine whether it is optimal to switch from your current state, say $B(t) = 0$, denoting no oil field development, to $B(t) = 1$, when oil field development is initiated. In stopping-rule problems, the decision-maker is monitoring the stochastic evolution of a state variable and waiting until the state variable reaches a threshold which says "Costly and irreversible investment is now optimal." In our models of the ANWR, the stochastic state variable will be the cost to a refinery of buying a barrel of crude oil ($/barrel). This price is referred to as "refiner acquisition cost." We wish to determine the price which would optimally trigger oil field development. We assume that if oil is extracted from the ANWR, that, in the minds of many people, the amenity dividend is irrevocably lost. It is not clear that the caribou and polar bear share this view, but in our analysis we will assume that a switch from B(t) = 0 to B(t) = 1 incurs not only the cost of development, production, and transport, but also the irreversible loss of amenity value. We will vary the size of the amenity loss from $200 million to $300 million per year to determine the sensitivity of the trigger price.[1]

A key question which must be addressed before determining the price which would trigger exploration and development of any oil beneath the ANWR is "How do oil prices evolve over time?" We will determine trigger prices for two alternative price processes. The model of geometric Brownian motion (GBM) assumes that the price of crude oil, P = P(t), evolves according to

$$dP = \mu P dt + \sigma P dz \tag{1}$$

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[1] There are approximately 100 million households in the U.S.. If each household were willing to pay two or three dollars per year to preserve a pristine ANWR, the amenity dividend would be $200 million or $300 million per year. Welsh and Poe (1998) report a median willingness-to-pay for modifying water releases from the Glen Canyon Dam to improve environmental conditions in the Grand Canyon to be at least $9.92 per household, per year. Thus, we do not regard an amenity dividend of $200 million to $300 million to be unrealistic.

where μ is called the mean drift rate, $\sigma > 0$ is the standard deviation rate, and $dz = \varepsilon(t)\sqrt{dt}$ is the increment of a Wiener Process, with $\varepsilon(t)$ being a standard normal variate. This process is referred to as a "continuous-time random walk with drift."

The other plausible price process is a mean-reverting (M-R) process which assumes that the price of crude oil evolves according to

$$dP = \eta(\overline{P} - P)dt + \sigma dz \qquad (2)$$

where $\overline{P} > 0$ is the mean or average price which prices tend to revert to over time, $\eta > 0$ is the speed of reversion, $\sigma > 0$ is the standard deviation rate and $dz = \varepsilon(t)\sqrt{dt}$ is, once again, the increment of a Wiener Process.[2]

GBM is a tractable stochastic process, and in our model, assuming that field development of the ANWR is irreversible, it will allow us to derive an analytic expression for the trigger price. This expression will depend on four parameters, describing field development and production, the discount rate, δ, μ and σ from Equation (1), and the size of the foregone amenity value, $A(t) = A$.

For the M-R process there is no analytic expression for the trigger price, but it is possible to numerically solve for the value function, which will depend on the current price for oil, P, the four field and production parameters, the discount rate, δ, η, $\overline{P}$, and σ from Equation (2) and amenity value, A. Examining the numerical values for the value

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[2] Alternative forms of the mean-reverting process include $dP = \eta(\overline{P} - P)Pdt + \sigma Pdz$ and $dP = \eta(\overline{P} - P)dt + \sigma Pdz$. See Dixit and Pindyck (1994) for a discussion. The specification in Equation (2) is used by Pindyck and Rubinfeld (1998) in their analysis of crude oil prices.

function, one can identify the price that triggers field development. We will vary amenity value, re-solve for the value function and the price trigger, and compare the results under the M-R process to those obtained under GBM.

Papers that view oil field development as a real option include Paddock *et al*. (1988) and Cortazar and Schwartz (1997). In both papers the perspective is that of an oil company currently holding or about to bid for the option to explore and develop oil underlying a tract of land or the continental shelf. In these papers the oil company earns no dividend while waiting, may face an expiry date (where the option reverts back to the government), and will pay royalties and a corporate income tax on the oil it extracts. Our perspective is that of the U.S. government trying to decide when it is in the country's interest to open the ANWR for leasing, development and production. We view the amenity flow from the pristine ANWR as a legitimate "social dividend."

In the next section we provide some background on the ANWR, identifying the technical parameters common to both the GBM and M-R models. In Section III we present the crude oil price data and estimate μ and σ, for GBM, and η, $\bar{P}$, and σ, for the M-R process. In Section IV we compute and compare the trigger prices for GBM and the M-R process. The paper concludes in Section V.

II The Arctic National Wildlife Refuge (ANWR)

The ANWR is located in the northeastern corner of Alaska and is bounded by the Canadian Yukon to the east and the Beaufort Sea to the north (see Figure 1). The ANWR coastal plain, where oil and gas development would take place, is about 100 miles across and 30 miles wide, an area slightly larger than the State of Delaware. In 1980, the Alaska

National Interest Lands Conservation Act actually designated the 1.5 million acre coastal plain as a study area; to be evaluated for its oil and gas potential. In 1998, the U.S. Geological Survey issued a report assessing the oil and gas potential for what is referred to as Federal Area 1002 of the ANWR. Area 1002 includes most of the ANWR coastal plain. The report states that the amount of oil beneath Area 1002 has a 0.95 probability of being at least 11.6 billion barrels and a 0.05 probability of being at least 31.5 billion barrels. The mean, or expected, oil *in situ* was estimated to be 20.7 billion barrels. The amount of *economically recoverable oil* will ultimately depend on recovery technology and the price of crude, both of which will be evolving during the economic life of the field. Attanasi (2002) estimates the economically recoverable oil from Area 1002 as ranging from zero barrels, if the price of oil is \$15.2 per barrel, to 6.3 billion barrels if the price of oil is \$30 per barrel. We interpret this analysis to imply that the *expected marginal production cost* will be about \$15 per barrel, and that for prices above \$27 per barrel, cumulative production is expected to exceed 6.0 billion barrels. In our notation we let $c = 15$ (\$/bbl) denote the marginal cost of production and we assume a field life of $\tau = 65$ years. Our estimate of average annual production is $Q = 100{,}000{,}000$ (bbl/year), for a cumulative production of $\tau Q = 6.5$ billion barrels. This estimate is smaller than the 9.12 billion barrels which the oil industry expects to produce if oil prices are in excess of \$24 per barrel. These parameter values are consistent with those reported by the McDowell Group (2002).

While the productive life for an ANWR oil field is expected to be $\tau = 65$ years, there will be a construction or field development phase expected to be $\lambda = 5$ years. During the construction phase, a present-value fixed cost of $K = \$2.9$ billion is expected.

We adopt an annual discount rate of $\delta = 0.10$ for the risk-adjusted annual rate for this project. The five parameters, c, τ, Q, λ, and δ, will be common to both the GBM and the M-R models, and their values are summarized in Table 1.

The other variable common to both models is the amenity value ($/year) which is assumed to be irrevocably lost if oil development takes place. We have no idea what this amount should be, so we will vary it between $200 million and $300 million per year to determine the sensitivity of trigger prices in both the GBM and M-R models. As noted earlier, this range for amenity value might correspond to a willingness to pay of two to three dollars per household per year.

If crude oil prices evolve according to GBM, we will also need estimates of μ and σ, the mean drift and standard deviation rates in Equation (1). If crude oil prices evolve according to a M-R process, we will need to estimate η, $\bar{P}$, and σ, the speed of reversion, mean to which prices revert, and standard deviation rate in Equation (2). In the next section we will present our time-series data for crude oil prices and describe how these process-specific parameters were estimated.

III. Estimation of μ and σ for GBM and η, $\bar{P}$, and σ for the M-R Process

Table 2 presents the price of crude oil paid by refiners for the years 1968 through 2001. These data are plotted in Figure 2. The prices are listed at the U.S. Department of Energy web site, http://www.eia.doe.gov/emeu/aer/txt/ptb0519.html This series contains 34 observations. Dixit and Pindyck (1994), in their discussion of mean-reverting processes, present a time-series of real crude oil prices spanning 120 years, from 1870 through 1990. Running a unit-root test for the full 120 years of data they reject GMB and fail to reject that the data were generated by a M-R process. They note, however, that if one does unit-root tests for the most recent 30 or 40 years, one cannot reject the hypothesis that the data were generated by GBM. This turned out to be the case for the data in Table 2. We could not reject GBM nor the M-R process, so we keep both models "in play," and present estimates of μ and σ for GBM in Equation (1) and for η, $\bar{P}$, and σ for the M-R process in Equation (2). These estimates are reported in Table 3.

The Maximum Likelihood Estimates of μ and σ for GBM can be obtained from the data in Table 2 by calculating the mean, m, and standard deviation, s, of the series $\ln(P_{t+1}/P_t)$ [see Reed and Clarke (1990), p. 149, for a discussion]. Then, $\hat{\mu} = m + s^2/2$ and $\hat{\sigma} = s$.

For the parameters of the M-R process, we ran the regression

$$P_{t+1} - P_t = a + bP_t + \varepsilon_t \tag{3}$$

and then calculated $\bar{P} = -\hat{a}/\hat{b}$, $\hat{\eta} = -\ln(1+\hat{b})$, and $\hat{\sigma} = \hat{\sigma}_\varepsilon \sqrt{-\hat{\eta}/[(1+\hat{b})^2 - 1]}$ (see Dixit and Pindyck (1994), pp. 76-77.). The regression results giving rise to the M-R parameter estimates in Table 3 are summarized in Table 4.

With the ANWR Field and Production Parameters from Table 1 and the estimates of the parameters for GBM and the M-R process in Table 3, we now determine the trigger prices which would make oil development optimal. When oil prices follow GBM, we can derive an analytic solution for the trigger price. When oil prices follow a M-R process we must numerically solve for the ANWR value function and then determine the trigger price.

IV. Trigger Prices under GBM and the M-R Process

We first consider the case where oil prices follow GBM. We need to determine two value functions; the value function for the ANWR while waiting to develop, which will be denoted by $V_W(P)$, and the value function when field development is started, which will be denoted by $V_F(P)$. Recall that there is a construction phase of $\lambda = 5$ years before oil production begins and that production is expected to last $\tau = 65$ years with an average output of Q = 100,000,000 barrels per year. Suppose when construction is initiated that the price of crude oil is P. With prices evolving according to GBM we expect the price to be $Pe^{\mu\lambda}$ when production starts λ years from the start of construction. Recall that μ is the expected drift rate in the price of crude oil. (We assume that the discount rate, δ, exceeds the expected drift rate for crude oil prices, so that $\delta > \mu$.) Then, the expected price at instant $t \geq \lambda$ would be $Pe^{\mu t}$. With a present value fixed cost for field development of K = \$2.9 billion, the expected present value of the ANWR field, when construction is initiated with an oil price of P, is given by

$$V_F(P) = -K + Q\int_{\lambda}^{\lambda+\tau} [Pe^{\mu t} - c]e^{-\delta t}dt \qquad (4)$$

Integrating yields the analytic expression

$$V_F(P) = -K + Qe^{-\delta\lambda}[(1-e^{-(\delta-\mu)\tau})Pe^{\mu\lambda}/(\delta-\mu) - (1-e^{-\delta\tau})(c/\delta)] \qquad (5)$$

As part of the smooth-pasting condition, to be discussed in a moment, we will need the expression for the derivative $V_F'(P)$ which is easily seen to be

$$V_F'(P) = Qe^{-\delta\lambda}(1-e^{-(\delta-\mu)\tau})[e^{\mu\lambda}/(\delta-\mu)] \qquad (6)$$

The value function while optimally waiting to develop, $V_W(P)$, must satisfy the *Hamilton - Jacobi - Bellman* (H-J-B) Equation which requires

$$\delta V_W(P) = A + \mu P V_W'(P) + (\sigma^2/2)P^2 V_W''(P) \qquad (7)$$

where A(t) = A is the constant amenity dividend provided by an undisturbed ANWR. The solution to this second-order ordinary differential equation is

$$V_W(P) = A/\delta + BP^{\beta} \qquad (8)$$

where B > 0 is an unknown constant, which will be determined using the value-matching and smooth-pasting conditions, and where $\beta > 1$ is the positive root of quadratic given by

$$\beta = (1/2 - \mu/\sigma^2) + \sqrt{(1/2 - \mu/\sigma^2)^2 + 2\delta/\sigma^2} \qquad (9)$$

The two terms on the right-hand-side of Equation (8) have the following interpretation. A/δ is the present value of the amenity dividend in perpetuity and is thus the value of *never* developing the ANWR's oil potential. BP^{β} is the value of the option to develop the ANWR's oil potential when the price of crude oil is currently P. B is an unknown, positive, constant which will be determined simultaneously with the as yet unknown trigger price.

At the trigger price, where one is indifferent between preserving a pristine ANWR and developing its oil potential, the value-matching condition requires $V_W(P) = V_F(P)$ or

$$A/\delta + BP^{\beta} = -K + Qe^{-\delta\lambda}[(1-e^{-(\delta-\mu)\tau})Pe^{\mu\lambda}/(\delta-\mu) - (1-e^{-\delta\tau})(c/\delta)] \qquad (10)$$

To ensure continuity when one switches from B(t) = 0 (no development) to B(t) = 1 (oil development), the smooth-pasting condition requires that $V_W'(P) = V_F'(P)$ at the trigger price, or

$$\beta BP^{\beta-1} = Qe^{-\delta\lambda}(1-e^{-(\delta-\mu)\tau})[e^{\mu\lambda}/(\delta-\mu)] \qquad (11)$$

The value-matching condition, Equation (10), and the smooth-pasting condition, Equation (11), constitute a two-equation system in the unknown constant B > 0, and the trigger price, P*. Some algebra leads to the following analytic expression for P*

$$P^* = \frac{[A + \delta K + e^{-\delta\lambda}Q(1-e^{-\delta\tau})c]\beta(\delta-\mu)}{\delta(\beta-1)e^{-(\delta-\mu)\lambda}Q(1-e^{-(\delta-\mu)\tau})} \qquad (12)$$

Knowing P*, you can determine B according to

$$B = \frac{e^{-(\delta-\mu)\lambda}(P^*)^{(1-\beta)}Q(1-e^{-(\delta-\mu)\tau})}{\beta(\delta-\mu)} \qquad (13)$$

Examination of Equation (12) reveals that the trigger price is linearly increasing in A, the amenity dividend of an undisturbed ANWR. We defer the reporting of the trigger prices under GBM until we discuss how they were computed when oil prices evolve according to the M-R process.

GBM, with $\mu = 0.04$, assumes that prices are expected to drift upward at a rate of 4% per year. In contrast to GBM, the M-R process assumes that prices tend to revert back to $\bar{P}$, and in our analysis of the oil prices in Table 2, we estimate that $\bar{P}$ = \$25.09 per barrel. One might expect that the trigger price would be higher if one thought oil prices were M-R, simply because one would not expect stochastic exponential growth for oil prices.

With oil prices evolving according to Equation (2), the expected net present value of field development, if development is initiated at price P, is given by $V_F(P)$ which now takes the form

$$V_F(P) = -K + Qe^{-\delta\lambda}[(1-e^{-\delta\tau})(\bar{P}-c)/\delta + (1-e^{-(\delta+\eta)\tau})(P-\bar{P})e^{-\eta\lambda}/(\delta+\eta)] \qquad (14)$$

$V_F(P)$ uses the result, when prices are M-R, that $E\{P(t)\,|\,P(0)\} = \bar{P} + (P(0) - \bar{P})e^{-\eta t}$. Because we won't be able to derive an analytic expression for P* when oil prices are M-R, we will use $V_F(P)$ to calculate boundary conditions for prices sufficiently high to induce field development. Note: $V_F(P)$ does not take into account any lost amenity value.

While optimally waiting to develop the ANWR field, the value function, $V_W(P)$, must satisfy an H-J-B Equation which now takes the form

$$\delta V_W(P) = A + \eta(\bar{P} - P)V_W{}'(P) + (\sigma^2/2)V_W{}''(P) \qquad (15)$$

There is no analytic solution for the $V_W(P)$ which satisfies Equation (15). We can solve a finite-time analogue to Equation (15), using the Implicit Finite Difference Method (IFDM), as discussed in Hull (2003), Wilmott et al. (1995) and Tavella and Randall (2000). The IFDM provides a numerical approximation for the value function that satisfies Equation (15). The problem is posed as a finite-horizon problem and the domain for time and price are in the bounded intervals $t \in [0, T]$ and $P \in [0, P_{MAX}]$. T must be sufficiently large so that the finite-horizon value function, evaluated at t=0, yields a good approximation to the infinite-horizon value function. P_{MAX} must be sufficiently high so that field development is optimal. For a finite-horizon, the H-J-B Equation, corresponding to Equation (15), becomes

$$\delta V^W(P,t) = A + V_t^W(P,t) + \eta(\bar{P} - P)V_P^W(P,t) + (\sigma^2/2)V_{PP}^W(P,t) \qquad (16)$$

where the superscript W now indicates that this is the value function while optimally waiting and the subscripts denote partial derivatives with respect to time, t, or price, P, since $V^W(P,t)$ depends on both price and time in the finite-horizon approximation.

Because there is no analytic solution, the value-matching and smooth-pasting conditions cannot be imposed directly. Instead, this problem may be formulated as a linear complementarity problem (LCP) [Wilmott et al. (1993)]. Define

$$HV \equiv \delta V^W(P,t) - \left(A + V_t^W(P,t) + \eta(\bar{P} - P)V_P^W(P,t) + (\sigma^2/2)V_{PP}^W(P,t)\right) \qquad (17)$$

The conditions for the LCP require (a) $HV \geq 0$, (b) $V^W(P,t) - V_F(P) \geq 0$, and (c) $HV \bullet \left[V^W(P,t) - V_F(P)\right] = 0$. Condition (a) says that $\delta V^W(P,t)$ must be greater than or equal to the sum of the dividend and the expected capital gain of the value function while waiting to develop. Condition (b) says that the value while waiting should not go below the expected net revenue from exercising the option now, $V_F(P)$. Condition (c) says that either (a) or (b) must hold as a strict equality. This implies that if HV = 0, then $V^W(P,t) - V_F(P) > 0$, and it is optimal to hold the option. If HV > 0, then $V^W(P,t) - V_F(P) = 0$ and it is optimal to exercise the option. Conditions (a) - (c) would be satisfied by a rational social planner holding the option to drill in the ANWR. When (a) - (c) hold, Friedman (1988), has shown that the value-matching and smooth-pasting conditions are satisfied.

Numerical solution, using the IFDM, requires that certain boundary and terminal conditions hold at $P = 0$, $P = P_{MAX}$, and $t = T$, respectively. These conditions require

$$V^W(0,t) = A/\delta \qquad \forall t \in [0,T] \qquad \text{(Boundary Condition One)}$$

$$V^W(P_{MAX},t) = V_F(P_{MAX}) \qquad \forall t \in [0,T] \qquad \text{(Boundary Condition Two)}$$

$$V^W(P,T) = \max\{V_F(P), A/\delta\} \qquad \forall P \in [0,P_{MAX}] \qquad \text{(Terminal Condition)}$$

Boundary Condition One says that when the price of oil is zero, the value function while waiting is the present value of the amenity dividend in perpetuity.[3] Boundary Condition Two says that at P_{MAX}, it is optimal to develop. The Terminal Condition says that at the expiry date (t=T) the value function is the argument that maximizes $[V_F(P), A/\delta]$.

The IFDM then seeks to determine the values of $V^W(P,t)$ that simultaneously satisfy the LCP, and the boundary and terminal conditions. The domain of the LCP is divided into a finite grid, $\{0,\Delta P,2\Delta P,\ldots,N\Delta P\}\times\{0,\Delta t,2\Delta t,\ldots,M\Delta t\}$, such that $N = P_{MAX}/\Delta P$ and $M = T/\Delta t$. There are several possible algorithms that might be used within the IFDM. We employ the *projected successive over-relaxation method* (PSOR) as discussed in Wilmott et al. (1995) and Tavella and Randall (2000)). Our MATLAB program, and additional details on the matrix notation for the PSOR Method are given in the Appendix.

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[3] Boundary Condition One is not strictly correct for the M-R process given by Equation (2). A zero oil price at instant t would result in a high probability of a positive price at instant $t+\Delta t$. The value function at P=0 would be greater than A/δ, reflecting the option to develop when price becomes sufficiently positive. If the option value is small, relative to A/δ, this will not have a significant effect on P*.

In this paper, we choose $P_{MAX} = \$50$ per barrel and $T = 100$ years, which are high and long enough to approximate the infinite-horizon problem. With $\Delta P = \$0.01$ and $\Delta t = 1$ (year) there are 505,101 interior, boundary, and terminal values that must be computed. The values for $V^W(P,t)$ are determined for A ranging from \$200,000,000 to \$300,000,000. The trigger prices for the M-R process are determined where $V_W(P, t)$ first coincides with $V_F(P)$, thus satisfying the LCP conditions. These trigger prices, along with the trigger prices previously calculated for GBM using Equation (12) are given in Table 5.[4]

For GBM, Equation (12) reveals that the trigger prices are linear in A and, given our estimates of the other parameters, vary from P*= \$19.84 per barrel when A=\$200 million per year, to P*=\$21.26 per barrel when A=\$300 million per year. As expected, the trigger prices under the M-R process are higher, ranging from P*=\$25.41 when A=\$200 million per year to P*=\$31.42 when A=\$300 million per year. Under GBM, the trigger prices were not particularly sensitive to the variation in A. The trigger prices under the M-R process were more sensitive to the variation in amenity value and, for $A > \$250$ million per year, development of the ANWR would not be optimal unless the price of oil was greater than \$28.29 per barrel.

The comparative statics of P* under GBM or the M-R process are summarized in Table 6. For GBM, an increase in c, δ, K, λ, or σ cause P* to increase, while an increase

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[4] The values for P_{MAX}, T, ΔP, and Δt would appear to provide a very close approximation to the values for the infinite-horizon problems when evaluated at t=0. We wrote MATLAB programs, using the PSOR algorithm, for both GBM and the M-R process. The numerical values obtained for GBM were identical, to the penny, to those obtained from the analytical expression for P* given by Equation (12).

in μ, Q, or τ cause P* to decrease. For the M-R process, an increase in c, δ, K, λ, or σ also cause P* to increase, while an increase in η, $\bar{P}$, Q, or τ cause P* to decrease.

V. Conclusions

This paper has determined the per barrel price for crude oil which would optimally trigger the exploration for oil in the coastal plain of the ANWR. The problem was posed as an optimal stopping problem where the price of crude oil evolved according to geometric Brownian motion or a mean-reverting process. For geometric Brownian motion, it was possible to derive an analytic solution for the trigger price. The trigger price, P*, was linear in amenity value, A, and for variations in A ranging from $200 million to $300 million per year the trigger price ranged from $19.84 per barrel to $21.26 per barrel.

If the crude oil prices follow a mean-reverting process, it is not possible to derive a closed-form solution for P*, but it is possible to numerically solve for the trigger price after approximating the value function. When A ranges from $200 million to $300 million per year, the price which would trigger exploration ranges from $25.41 per barrel to $31.42 per barrel, respectively.

The expected present net revenue of an ANWR field producing 6.5 billion barrels over a 65 year horizon will depend on the stochastic price process and the price when development is initiated. The expected present net revenue of the ANWR field is higher under geometric Brownian motion than under a mean-reverting process. For example, under geometric Brownian motion, and with an initial price of $24 per barrel, the expected present net revenue is $17.049 billion. Under a mean-reverting process, for the same initial price of \$24 per barrel, the 6.5 billion barrel field is only worth \$3.115 billion. The expected present net revenue under the mean-reverting process is smaller because our estimate of $\bar{P}$ was \$25.09 per barrel, based on data for the period 1968 - 2001.

The estimates of expected present net revenue, $V_F(P)$, do not take into account the present value of the foregone amenity value. We do not know what the annual amenity value is for an undisturbed (pristine) ANWR. We assumed that whatever it's value, that it would be irrevocably lost if oil development takes place. Some claim that the caribou and polar bear will be unaffected by production facilities and pipelines, and point to our experience with the production facilities at Prudhoe Bay and the Trans-Alaska Pipeline. The Central Arctic Caribou Herd, which calves in the Prudhoe Bay and Kuparuk oil fields is currently estimated at 23,400 animals, having increased from 3,000 animals during the four decades of development and production from Prudhoe Bay. We opted to take the perspective of a conservationist who would view the production facilities and pipeline as an irreversible violation of the arctic wilderness, even if the caribou, bear, and other wildlife might be more tolerant.

Finally, our analysis takes no account of the potential negative externalities from the production and transport of ANWR crude, nor the negative externalities that might be generated from the consumption of the resulting distillate products by Americans driving SUVs. The possibility of a tanker spilling ANWR oil in Southern Alaska, or the contribution of that oil to smog in Los Angles, or global warming, is difficult to assess. Such negative externalities might be incorporated in the models in this paper by increasing "c" from \$15 per barrel to some higher value to reflect the marginal "social" cost per barrel. This would cause an increase in P* and lower the expected discounted *social* net revenues from field development under either geometric Brownian motion or a mean-reverting process.

Appendix

First we discretize Equation (16) to apply the IFDM. Let indices i and j be $i \in \{0,1\ldots,M\}$ and $j \in \{0,1,\ldots,N\}$, such that $M = T/\Delta t$ and $N = P_{MAX}/P$. Now the domain is a finite grid of dimension $\{0,\Delta t,2\Delta t,\ldots,M\Delta t\} \times \{0,\Delta P,2\Delta P,\ldots,N\Delta P\}$. Next, we define $V_{i,j}$ as the value at point $(i\Delta t, j\Delta P)$ such that $V_{i,j} = V(i\Delta t, j\Delta P)$. Now let $V_W(i\Delta t, j\Delta P) = V_{i,j}^{w}$ and $V_F(j\Delta P) = V_j^F$.

Equation (16) is discretized as

$$\delta V_{i,j}^W = A + \frac{V_{i+1,j}^W - V_{i,j}^W}{\Delta t} + \eta(\overline{P} - j\Delta P)\left(\frac{V_{i,j+1}^W - V_{i,j-1}^W}{2\Delta P}\right) + \frac{\sigma^2}{2}\left(\frac{V_{i,j+1}^W + V_{i,j-1}^W - 2V_{i,j}^W}{\Delta P^2}\right) \quad (A.1)$$

The boundary and terminal conditions are

$$V_{i,0} = A/\delta \qquad \forall i \in \{0,1,\ldots,M\} \quad (A.2)$$

$$V_{i,N} = V_N^F = V^F(P_{MAX}) \qquad \forall i \in \{0,1,\ldots,M\} \quad (A.3)$$

$$V_{M,j} = \max\left\{V_j^F, A/\delta\right\} \qquad \forall j \in \{1,2,\ldots,N-1\} \quad (A.4)$$

Expanding Equation (A.1) and collecting the terms for $V_{i,j}^W$, $V_{i+1,j}^W$, $V_{i,j-1}^W$ and $V_{i,j+1}^W$ yields

$$dV_{i,j}^{W} + e_j V_{i,j-1}^{W} + f_j V_{i,j+1}^{W} - A\Delta t = V_{i+1,j}^{W} \qquad \forall j \in \{1,2,\ldots,N-1\} \tag{A.5}$$

where

$$d = 1 + \delta\Delta t + \frac{\Delta t \sigma^2}{\Delta P^2} \tag{A.6}$$

$$e_j = -\Delta t\left(\frac{\sigma^2}{2\Delta P^2} - \frac{\eta(-j\Delta P + \overline{P})}{2\Delta P}\right) \tag{A.7}$$

and

$$f_j = -\Delta t\left(\frac{\sigma^2}{2\Delta P^2} + \frac{\eta(-j\Delta P + \overline{P})}{2\Delta P}\right) \tag{A.8}$$

The system represented by Equation (A.5) can also be expressed in matrix form. Define the matrix B and the vectors $\overline{A}$, x_i and b_i as

$$B = \begin{bmatrix} d & f_1 & 0 & \cdots & \cdots & 0 \\ e_2 & d & f_2 & 0 & \cdots & \vdots \\ 0 & e_3 & d & f_3 & \ddots & \vdots \\ \vdots & 0 & e_4 & d & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & f_{N-2} \\ 0 & \cdots & \cdots & 0 & e_{N-1} & d \end{bmatrix}, \quad \overline{A} = \begin{bmatrix} A\Delta t \\ A\Delta t \\ \vdots \\ A\Delta t \end{bmatrix}$$

$$x_i = \begin{bmatrix} V_{i,1} \\ V_{i,2} \\ \vdots \\ V_{i,N-2} \\ V_{i,N-1} \end{bmatrix}, \quad b_i = \begin{bmatrix} V_{i,1} - e_1 V_{i-1,0} \\ V_{i,2} \\ \vdots \\ V_{i,N-2} \\ V_{i,N-1} - f_{N-1} V_{i,N} \end{bmatrix}.$$

where B is an $(N-1)\times(N-1)$ matrix and $\overline{A}$, x_i and b_i are $(N-1)\times 1$ vectors.

Given the above matrix and vectors, Equation (A.5) may now be written

$$Bx_i - \overline{A} = b_{i+1} \qquad \forall i \in \{0,1,\ldots,M-1\} \tag{A.9}$$

The LCP, for each i, may now be formulated as (a) $Bx_i - \overline{A} - b_{i+1} \geq 0$, (b) $x_i \geq g$, and (c) $\left(Bx_i - \overline{A} - b_{i+1}\right)\left(x_i - g\right) = 0$, where

$$g = \begin{bmatrix} V_1^F \\ V_2^F \\ \vdots \\ V_{N-1}^F \end{bmatrix}. \tag{A.10}$$

Given the boundary and terminal conditions, (A.9) can be solved for x_{M-1} using the PSOR Method [Wilmott et al. (1995) or Tavella and Randall (2000)], since the system consists of N-1 equations with N-1 unknowns. After obtaining x_{M-1}, the system can be solved for x_{M-2}, and so on. Backward calculations will be done until x_0, the vector of values at t=0, is obtained.

The term $Bx_i - \overline{A} = b_{i+1}$ is solved by generating a Cauchy sequence $\left\{ x_i^{(k)} \right\}$ according to

$$V_{i,j}^{(k)} = \frac{V_{i+1,j} + A\Delta t - e_j V_{i,j-1}^{(k)} - f_j V_{i,j+1}^{(k-1)}}{d} \qquad \forall j \in \{1,2,\ldots,N-1\} \qquad \text{(A.11)}$$

where superscript k is the number of iterations. The iteration process continues until $\left\| x_i^{(k)} - x_i^{(k-1)} \right\|_2^2 < \varepsilon$ where ε is a sufficiently small value; in our case, $\varepsilon = 10^{-8}$. The solution of the system of simultaneous equations is the one that satisfies the LCP and $\left\| x_i^{(k)} - x_i^{(k-1)} \right\|_2^2 < \varepsilon$ at the same time. The logic of the PSOR algorithm is as follows:

```
while error $> \varepsilon$
    for $j = 1 : n$,
        $\overline{V}_{i,j}^{(k)} = \frac{V_{i+1,j} + A\Delta t - e_j V_{i,j-1}^{(k)} - f_j V_{i,j+1}^{(k-1)}}{d}$
        $\overset{f}{V}_{i,j}^{(k)} = V_{i,j}^{(k-1)} + \omega\left(\overline{V}_{i,j}^{(k)} - V_{i,j}^{(k-1)}\right)$
        $V_{i,j}^{(k)} = \max\left\{ \overset{f}{V}_{i,j}^{(k)}, V_j^F \right\}$
    end
    error $= \left\| x_i^{(k)} - x_i^{(k-1)} \right\|_2^2$
end
```

where $\omega \in \left(1,2\right)$ is the "over-relaxation parameter."

The above algorithm solves the LCP and gives us all the $V_{i,j}$ on the domain. Accordingly, we can identify the point at which field development is optimal at each time step. The trigger prices under the M-R process, at $t = 0$, are reported in Table 5 along with the trigger prices for GBM as per Equation (12).

The MATLAB code for our M-R PSOR algorithm is given below. We were extremely fortunate have the assistance of John Zollweg and Linda Buttel, both at the Cornell Theory Center. John took our original MATLAB program, which was taking days to produce the P* values for the M-R process, and, through his programming wizardry, streamlined it solve for the P* values in 25 minutes. Linda is currently converting the program to C for additional solution speed and will be developing a Windows interface to input parameter values, display output, and explore comparative statics. Thank you John and Linda!

```
function F=ANWRMR
[vars,names]=xlsread('.\anwrmr.xls');
delta=vars(strmatch('delta',names,'exact'));
sigma=vars(strmatch('sigma',names,'exact'));
eta=vars(strmatch('eta',names,'exact'));
T=vars(strmatch('T',names,'exact'));
M=vars(strmatch('M',names,'exact'));
N=vars(strmatch('N',names,'exact'));
Pmax=vars(strmatch('Pmax',names,'exact'));
Pbar=vars(strmatch('Pbar',names,'exact'));
omega=vars(strmatch('omega',names,'exact'));
tol=vars(strmatch('tol',names,'exact'));
tau=vars(strmatch('tau',names,'exact'));
lambda=vars(strmatch('lambda',names,'exact'));
A=vars(strmatch('A',names,'exact'));
C=vars(strmatch('C',names,'exact'));
K=vars(strmatch('K',names,'exact'));
Q=vars(strmatch('Q',names,'exact'));
r=vars(strmatch('r',names,'esact'));
time0=clock;
F=zeros(N+1,r);
dt=T/M;
dp=Pmax/N;
Tdist=dt*(0:1:M);
```

```
Pdist=dp*(0:1:N);
e=-dt*((sigma^2/(2*dp^2))-eta*(-Pdist(2:N)+Pbar)/(2*dp));
d=1+dt*delta+dt*sigma^2/(dp^2);
f=-dt*(sigma^2/(2*dp^2)+eta*(-Pdist(2:N)+Pbar)/(2*dp));
g0=-K+Q*exp(-delta*lambda)*((1-exp(-delta*tau))*(Pbar-C)/delta+(1-exp(-(eta+delta)*tau)) *((Pdist-Pbar)*exp(-
eta*lambda)/(eta+delta)));
c=g0(N+1);
for k=r:-1:1
 A=(k-1)*10000000+200000000;
 a=A/delta;
 Value=zeros(N+1,M+1);
 Value(:,M+1)=max(g0,a)';
 g=max(g0(2:N),a);
 for j=M:-1:1
  if j==M
   b(1:N-1)=g;
   x=g;
  else
   b(1:N-1)=Value(2:N,j+1);
   if j==M-1
    x=Value(2:N,j+1)';
   else
    if j<M-3
     adj=min(mean((Value(2:nold,j+2)-Value(2:nold,j+1))./(Value(2:nold,j+3)-Value(2:nold,j+2))),1.01*adj);
    else
     adj=1;
    end
    nold=n;
    x=((1+adj)*Value(2:N,j+1)-adj*Value(2:N,j+2))';
   end
  end
  b(1)=b(1)-e(1)*a;
  b(N-1)=b(N-1)-f(N-1)*c;
  [x,n]=KojiPSOR(x,e,d,f,g,b,omega,tol,A,dt);
  Value(1,j)=a;
  Value(2:N,j)=x';
  Value(N+1,j)=c;
 end;
 F(:,k)=Value(:,1);
end;
fprintf('Anwrmr took %9.3d sec.\n',etime(clock,time0));
function [x,n]=KojiPSOR(x,e,d,f,g,b,omega,tol,A,dt)
Z=length(x);
err=100;
xbar=zeros(1,Z);
count=0;
time0=clock;
y=(b+A*dt)./d;
ed=e./d;
fd=f./d;
while err>tol
  ybar=y(1)-fd(1)*x(2);
  xbar(1)=x(1)+omega*(ybar-x(1));
  if g(1)>xbar(1)
    xbar(1)=g(1);
  end
  for n=2:Z-1
    ybar=y(n)-fd(n)*x(n+1)-ed(n)*xbar(n-1);
    xbar(n)=x(n)+omega*(ybar-x(n));
    if g(n)>xbar(n)
      xbar(n)=g(n);
      if n>0.45*Z & count>1
```

```
               break
            end
        end
    end
    ybar=y(Z)-ed(Z)*xbar(Z-1);
    xbar(Z)=x(Z)+omega*(ybar-x(Z));
    if g(Z)>xbar(Z)
        xbar(Z)=g(Z);
    end
    err=sum((xbar-x).^2);
    count=count+1;
    if err<1e-4
        omega=1.2;
    end
    x=xbar;
end
fprintf('%5i iterations in PSOR took %7.3f sec.; estimated P* = $%5.2f\n', count,etime(clock,time0),0.01*n);
```

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Figure 1. A Map of Alaska and the ANWR

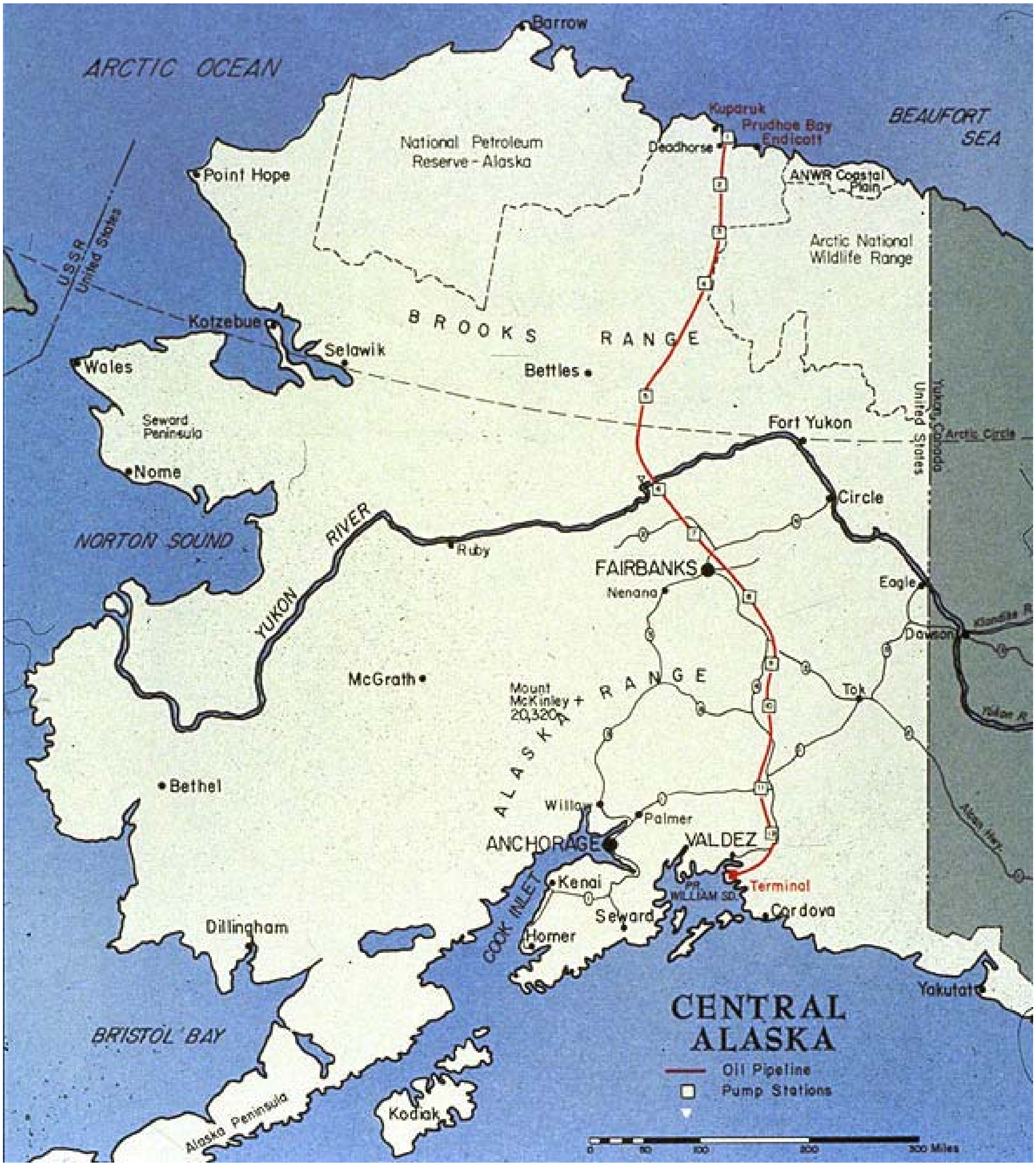

Figure 2. The Price of Crude Oil, 1968-2001.

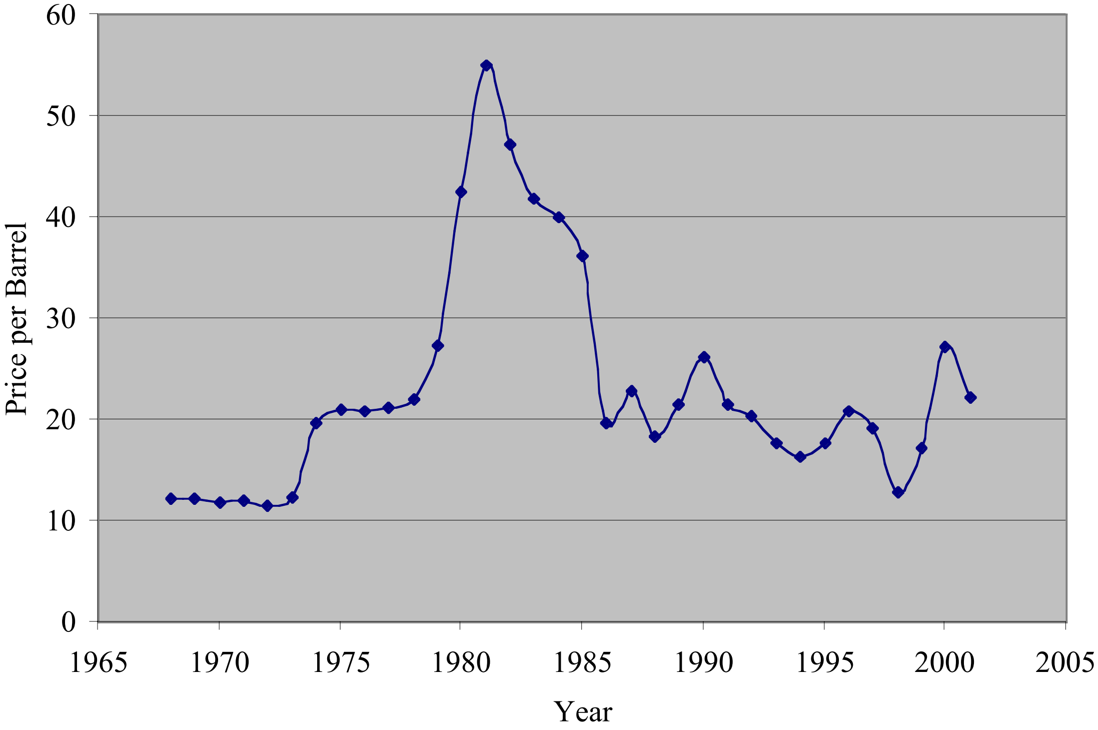

Table 1. ANWR Field and Production Parameters

Parameter	Value
$c =$	15 (marginal cost, \$/barrel)
$\tau =$	65 (years of production)
$Q =$	100,000,000 (average annual production)
$\lambda =$	5 (construction phase, years)
$\delta =$	0.1 (annual discount rate)

Table 2. Crude Oil Prices, 1968 - 2001, Paid by Refiners ($/Barrel)

Year	Price
1968	12.21
1969	12.21
1970	11.91
1971	12.06
1972	11.53
1973	12.41
1974	19.61
1975	20.96
1976	20.9
1977	21.21
1978	22
1979	27.31
1980	42.48
1981	55.04
1982	47.12
1983	41.91
1984	39.94
1985	36.18
1986	19.68
1987	22.89
1988	18.38
1989	21.46
1990	26.11
1991	21.56
1992	20.29
1993	17.72
1994	16.32
1995	17.67
1996	20.77
1997	19.23
1998	12.77
1999	17.1
2000	27.2
2001	22.25

Table 3. Estimates of μ and σ for GBM and η, $\bar{P}$, and σ for the M-R Process

GBM, Equation (1)	
$\hat{\mu} =$	0.04
$\hat{\sigma} =$	0.05
M-R Process, Equation (2)	
$\hat{\eta} =$	0.18
$\hat{\bar{P}} =$	25.09 ($)
$\hat{\sigma} =$	4.5 ($)

Table 4. Regression Results for the M-R Process in Equation (3)

Coefficient or Statistic	Value (t−statistic)
$\hat{a}$	4.07 (1.71)
$\hat{b}$	−0.16 (−1.75)
σ_ε	5.79
Adjusted R^2	0.06

Table 5. Trigger Prices for GMB and the M-R Process.

A (Amenity Value, \$/year)	P* (GBM, \$/barrel)	P* (M-R, \$/barrel)
200,000,000	19.84	25.41
210,000,000	19.99	25.97
220,000,000	20.13	26.54
230,000,000	20.27	27.12
240,000,000	20.41	27.70
250,000,000	20.55	28.29
260,000,000	20.69	28.90
270,000,000	20.84	29.51
280,000,000	20.97	30.14
290,000,000	21.12	30.77
300,000,000	21.26	31.42

Table 6. Comparative Statics of P* under GBM and the M-R Process

Price Process	c	δ	η	K	λ	μ	$\bar{P}$	Q	σ	τ
GBM	+	+	n.a.	+	+	-	n.a.	-	+	-
M-R	+	+	-	+	+	n.a.	-	-	+	-

n.a. = not applicable