Title: Renewable Resource Management

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Summary: The management of renewable resources can be viewed as a dynamic allocation problem. How much of a resource should be harvested today and how much should be left for tomorrow? Models of resource management might partition time into discrete, uniform intervals (for example, years), or they may treat time as continuous. Resource growth might be deterministic or stochastic (where a random variable or process influences the evolution of the resource stock). This essay looks at the four types of renewable resource models which might result from discrete- or continuous-time coupled with deterministic or stochastic dynamics. After presenting four reasonably general models, six specific models, applied to fishery, forest, and groundwater resources, are presented. The Method of Lagrange Multipliers, the Maximum Principle, and Dynamic Programming are used to determine optimal allocation or the form of an optimal, adaptive policy. The four general models and the six specific models were designed to give the reader all the necessary theory and methods to confidently approach the large and growing literature on the economics of renewable resources. In the process of working through these models, the reader should also gain an understanding of steady-state equilibria in deterministic models and adaptive policies and stationary distributions in stochastic models, and how these concepts might relate to the often ill-defined term, "sustainable resource use." The essay concludes with a discussion of some of the impediments to improved resource management, the information needed to estimate the parameters of renewable resource models, and the institutions that might be needed to improve the allocation of fishery, forest, and water resources.

1. Introduction

This essay will present some basic economic models of renewable resources. A renewable resource is one which exhibits significant growth or renewal over an economic horizon. Most plant and animal populations would be regarded as renewable resources. The water in a lake, stream, or underground aquifer, if replenished through a cycle of evaporation and precipitation, might also be regarded as a renewable resource. If, however, a resource has a very small rate of growth or renewal, it might be more appropriate, from an economic perspective, to regard it as a nonrenewable resource. For example, the remaining stands of old-growth coast redwood (*Sequoia sempervirens*), found in California, may be 1,000 years old or older. While redwoods can be cultivated, the length of time to achieve old-growth status may be so long, relative to most economic planning horizons, that these majestic trees are best regarded as a nonrenewable resource. Similarly, some aquifers have such a small rate of recharge that they are more like an underground pool of oil, and thus more appropriately modeled as a nonrenewable resource. Because renewable resources exhibit a significant rate of growth or renewal they would seem good candidates for sustainable harvest. The definition of sustainability is problematic. This is particularly the case if we admit that the rate of growth or renewal for a resource fluctuates through time. Indeed, the process of evolution raises some fundamental questions about the feasibility of sustainable resource use.

The remainder of this introductory section will present the components of the basic bioeconomic model. The subsection on resource dynamics will introduce the distinction between continuous- and discrete-time models as well as deterministic and stochastic models. The second subsection formulates general objectives for resource management within a deterministic and stochastic environment. The third subsection discusses sustainability and adaptive management. Section 2 will assemble the components from subsections 1.1 and 1.2 and present four bioeconomic models,

Section 3 contains six models of fishery, forest, and groundwater resources. These models are special cases of the more general models of Section 2. These models give further insight into the basic problem of resource management. Section 4 concludes with a discussion of the practicality of these models and the impediments to improved resource management in the real world.

1.1 Resource Dynamics

Resource economics is concerned with how natural resources are allocated over time. The stock of a renewable resource will change with net natural growth and harvest. The change in a resource stock might be modeled using a differential or difference equation. Let X_t denote the stock of a renewable resource at instant t and $X_{t+\Delta t}$ the stock at instant $t+\Delta t$, where we initially assume Δt is some small but positive increment of time. Suppose over the interval Δt that the net natural rate of growth is given by the function $F(X_t)$ and that the level of harvest is denoted by $Y_t \geq 0$. Assume that X_t, $F(X_t)$ and Y_t are all measured in the same units. (In the models of this essay we will assume that the mass or volume of a resource stock can be measured, for example metric tons of herring, cubic meters of wood, or gallons of water.) Then the rate of change in the resource, going from t to $t+\Delta t$, may be calculated according to

$$\frac{X_{t+\Delta t} - X_t}{\Delta t} = F(X_t) - Y_t \tag{1}$$

If time is continuous we can let $\Delta t \to 0$ and equation (1) becomes a differential equation that is often written as

$$\dot{X} = F(X) - Y \tag{2}$$

where $\dot{X} = dX/dt$ denotes the time rate of change in the resource and X and Y are the resource stock and level of harvest at instant t, respectively. Alternatively, if $\Delta t=1$ equation (1) becomes the first-order difference equation

$$X_{t+1} - X_t = F(X_t) - Y_t \tag{3}$$

Equation (3) is often written in *iterative form* as

$$X_{t+1} = X_t + F(X_t) - Y_t = G(X_t, Y_t) \tag{4}$$

If X_0 and Y_0 are known then $X_1 = G(X_0,Y_0)$. If Y_1 is known then $X_2 = G(X_1,Y_1)$, and one could *simulate* the dynamics of the stock for a known or candidate harvest schedule, Y_t, from the initial condition, X_0. Modern spreadsheet software makes such simulations relatively easy to do.

A stochastic or fluctuating environment may make growth, and thus the stock in period t+1, a random variable. Suppose z_{t+1} is a random variable, perhaps water temperature, that influences the growth of a fish stock. The realized value for z_{t+1} can only be observed at the beginning of t+1. The value of X_{t+1} is determined by

$$X_{t+1} = G(X_t, Y_t; z_{t+1}) \tag{5}$$

Suppose in period t we can observe or accurately measure X_t. While X_t is observable, the consequences of Y_t on X_{t+1} cannot be known with certainty in period t, when a decision on the level of Y_t must be made. It is usually assumed that the random variables, z_{t+1}, are independent and identically distributed (i.i.d.), being generated by the density function $f(z_{t+1})$.

Stochastic differential equations are also used in modeling the dynamics of natural resources. In such models the resource stock becomes an *Itô variable* with dynamics described by

$$dX = [F(X) - Y]dt + \sigma(X)dz \tag{6}$$

The first term on the right-hand-side of (6) is called *the mean or expected drift rate*. It depends on the relative rates for net growth and harvest, as in the ordinary differential equation, (2). The term $\sigma(X) > 0$ is *the standard deviation rate,* and $dz = \varepsilon(t)\sqrt{dt}$ is the increment of a *Wiener process,* where $\varepsilon(t)$ is a standard normal random variable, $\varepsilon(t) \sim N(0,1)$. In Sections 2 and 3 of this essay we will consider dynamic optimization problems based on the above equations for deterministic and stochastic growth. The deterministic models are generally more tractable and might be solved using the Method of LaGrange Multipliers, the Calculus of Variations, or the Maximum Principle. Problems with stochastic growth [equations (5) and (6)] will typically employ Dynamic Programming to find an optimal harvest policy.

1.2 Management Objectives

A well-defined resource management problem needs a clear objective. There are many potential objectives. A reasonably general approach is to define $\pi_t = \pi(X_t, Y_t)$ to be the net benefits at instant or period t from having a resource stock of size X_t and harvest at rate Y_t. In continuous-time models, this objective is often written as $\pi = \pi(X,Y)$, with the presumption that π, X, and Y are all measured at instant t.

It is possible that net benefits might only depend on the rate of harvest, in which case $\pi = \pi(Y)$. Dependence of net benefits on the resource stock can arise for at least two reasons. First, in a strictly commercial setting, the cost of harvesting Y at instant t may depend on the size of the stock at instant t. It is the case for many resources that the larger the stock, the lower the cost for any level of harvest. Second, for certain animals, most notably marine mammals, the stock may convey "non-consumption" benefits associated with wildlife observation. The larger the stock of, say, humpback whales, the more likely they will be seen by humans on a "whale-watch cruise." For certain species, humans may derive an "existence value" simply knowing that the population still exists in the wild. Larger populations may mean the species is more secure, and existence benefits may be higher.

Underlying $\pi = \pi(X,Y)$ is the presumption that "economic man is the measure of all value." Such a perspective does not prevent *homo economicus* from having environmental and conservation motives. To determine their importance, nonconsumptive benefits must be estimated, in a dollar metric, so that the value of a larger stock in the future can be compared with the increment in benefits that might be obtained from a larger harvest today. This cuts to the heart of resource management; the need determine the "best" harvest schedule from many feasible schedules. Different harvest schedules will have different implications for resource dynamics and the future flow of net benefits. Calculating the present value of net benefits is one way to rank or evaluate alternative harvest schedules. In continuous time the present value, PV, of net benefits from a harvest schedule, Y, that induces the resource trajectory, X, over an infinite horizon is given by

$$PV = \int_0^\infty \pi(X,Y)e^{-\delta t}dt \tag{7}$$

where $e^{-\delta t}$ is the continuous-time discount factor and $\delta > 0$ is the instantaneous rate of discount. A common objective in bioeconomics is to maximize the present value of net benefits with respect to the harvest schedule, Y, for $\infty > t \geq 0$.

In discrete time, where the net benefits in period t are $\pi_t = \pi(X_t,Y_t)$, the present value of net benefits is written as

$$PV = \sum_{t=0}^{\infty} \rho^t \pi(X_t, Y_t) \tag{8}$$

where $\rho = 1/(1 + \delta)$ is the discrete-time discount factor and δ is now the periodic rate of discount. As in continuous time, a common objective is to maximize PV by choosing $Y_t \geq 0$ for $t = 0,1,2,...,\infty$.

When growth is stochastic the objective of resource management is often the maximization of *expected* present value. Dynamic programming is used to find a *value function* that gives the expected present value in period t from having a stock of size X_t, *assuming that the resource is optimally harvested in the future.* The value function, $V_t(X_t)$ must satisfy a recursive equation, called the *Bellman equation,* which takes the form

$$V_t(X_t) = \text{Max}[\pi(X_t, Y_t) + \rho E_t\{V_{t+1}(G(X_t, Y_t; z_{t+1}))\}] \tag{9}$$

where $E_t\{\cdot\}$ is the expectation operator in period t and the maximization of $[\cdot]$ is with respect to Y_t. The value function, $V_t(X_t)$ requires that Y_t be chosen so as to maximize the sum of current net benefits, $\pi(\cdot)$, plus the discounted expected value of having a stock size of $X_{t+1} = G(X_t,Y_t;z_{t+1})$ in period t+1. If X_t and Y_t are continuous variables, if $\pi(\cdot)$, $V_{t+1}(\cdot)$ and $G(\cdot)$ are concave, differentiable functions, and if the expectation operation is well defined, then the *maximal condition* requires $\partial[\cdot]/\partial Y_t = 0$, or

$$\partial\pi(\cdot)/\partial Y_t + \rho\,\partial E_t\{V_{t+1}(G(X_t, Y_t; z_{t+1}))\}/\partial Y_t = 0 \tag{10}$$

Equation (10) is a single equation in X_t and Y_t and will imply the *optimal feedback policy*

$$Y_t = \varphi_t(X_t) \tag{11}$$

In many infinite-horizon problems the value function and optimal harvest policy are *stationary*, meaning that they don't depend on time. In this case $Y_t = \varphi(X_t)$ and the maximized expected present value of having a stock of size X_t is

$$V(X_t) = \pi(X_t, \varphi(X_t)) + \rho E_t\{V(G(X_t, \varphi(X_t); z_{t+1}))\} \tag{12}$$

where substitution of the optimal feedback policy has accomplished the maximization required in expression (9) and the expectation operator will "integrate out" z_{t+1}, leaving $V(X_t)$.

To summarize, given forms for $\pi(X_t,Y_t)$, $G(X_t,Y_t;z_{t+1})$, the probability density $f(z_{t+1})$, and the discount factor $\rho = 1/(1 + \delta)$, dynamic programming is used in an attempt to find the value function, $V_t(X_t)$ and the feedback or adaptive harvest policy, $Y_t = \varphi_t(X_t)$, that will maximize the expected present value of net benefits.

1.3 Sustainability and Adaptive Management

The management of a renewable resource in a deterministic environment might result in a steady-state equilibrium where $X_t = X_{t+1} = X \geq 0$ and $Y_t = Y_{t+1} = Y \geq 0$. The steady-state equilibrium, (X,Y), is also called a fixed point, since it is a point in X-Y space which will perpetuate itself. In the continuous- or discrete-time equations for resource dynamics [equations (2) and (3)] a steady state equilibrium must satisfy Y = F(X). In words, a steady state is characterized by harvest equaling net growth. This makes intuitive sense, since, when harvest equals net growth, stock is unchanging. The net growth function is often specified so that $F(X) > 0$ for $0 < k < X < K$, with $F'(X) > 0$ for $k \leq X < X_{msy}$ and $F'(X) < 0$ for $X_{msy} < X \leq K$, where X_{msy} is the stock size where F(X) reaches a maximum and $Y_{msy} = F(X_{msy})$ is the maximum sustainable yield. A net growth function with these attributes is the cubic function

$$Y=F(X)=rX(X/k-1)(1-X/K) \tag{13}$$

shown in Figure 1, where r=1, k=0.25, K=1, and it can be shown that X_{msy}=0.717.

(Figure 1. about here)

For the net growth function in Figure 1, r > 0 is called the intrinsic growth rate, k > 0 is called the minimum viable population, and K > 0 is called the environmental carrying capacity.

For k < X < K, any point on the net growth function, F(X), is a steady state, and therefore an equilibrium which can support a *sustainable harvest* Y = F(X) > 0. To identify a preferred steady state equilibrium will required the specification of an objective which can be used to rank the infinite number of combinations (X,Y) which would support a sustainable harvest. Depending on management objectives, the preferred steady state might be at a stock size greater than or less than X_{msy}.

In some deterministic models and almost all stochastic models, a sustainable harvest, where $Y_t = Y > 0$, will not be desirable. For stochastic models, where the stock is an induced random variable, a constant harvest policy is not optimal. Intuitively, if the stock is being bounced around by stochastic environmental factors, you would wish to harvest it in an adaptive way, where it is optimal to harvest more of a resource in a year when growth was greater than expected, and less in a year when growth was less than expected.

2. Four Bioeconomic Models

In this section four bioeconomic models will be presented. These models will combine the elements introduced in Section 1 and set the stage for some of the special cases in Section 3.

2.1 A Continuous-Time, Deterministic Model

The continuous-time, deterministic model of this subsection seeks to

$$\text{Maximize } PV = \int_0^{\infty} \pi(X,Y)e^{-\delta t}dt$$
$$\text{Subject to } \dot{X} = F(X) - Y, X(0) \text{ given} \qquad \text{(P1)}$$

In words, the resource manager seeks the harvest schedule, Y = Y(t), which will maximize the present value of net benefits subject to the equation describing resource dynamics and a given initial stock level, X(0). From this perspective, resource management becomes a dynamic optimization problem. The most convenient way to solve (P1) is to use the maximum principle and formulate the current-value Hamiltonian

$$H = \pi(X,Y) + \mu[F(X) - Y] \qquad (14)$$

where $\mu = \mu(t)$ is the current-value shadow price on the resource stock, *in situ*, at instant t. The current-value Hamiltonian can be thought of as the rate of increase in the asset value of the resource stock at instant t. It is comprised of the net benefit flow, $\pi(X,Y)$, and a capital gain term, $\mu[F(X) - Y]$. If H is maximized, the present value of net benefits in (P1) will be maximized. The optimal trajectories for Y, X and μ must satisfy a set of equations (first-order conditions) involving partial derivatives of H. If H is concave in X and Y these first-order conditions will be necessary and sufficient. Note: If H is concave in X and Y, it can be shown that the maximized Hamiltonian is concave in X. The first-order conditions require

$$\partial H/\partial Y = \partial\pi(\cdot)/\partial Y - \mu = 0 \qquad (15)$$

$$\dot{\mu} - \delta\mu = -\partial H/\partial X = -[\partial\pi(\cdot)/\partial X + \mu F'(X)] \qquad (16)$$

$$\dot{X} = \partial H/\partial\mu = F(X) - Y \qquad (17)$$

In addition to the necessary conditions (15) - (17), there is the initial condition that X(0) is given and what is called the *transversality condition*, which for this problem requires

$$\lim e^{-\delta t}\mu(t)X(t) \to 0 \text{ as } t \to \infty \qquad (18)$$

It turns out that the transversality condition is required for the integral in (P1) to converge and for the optimization problem to be well-defined.

The qualitative analysis of the solution to (P1) might proceed as follows: (a) take a time derivative of the maximal condition, equation (15), and solve for $\dot{\mu}$, (b) substitute the expressions for $\dot{\mu}$ and μ into the co-state equation, equation (16), and solve for an expression for $\dot{Y}$, (c) identify the *isoclines* for the $\dot{X}, \dot{Y}$ dynamical system (the points where the isoclines intersect are steady-state equilibria), and (d) identify the *directionals* or *vector field* in each *isosector* and try to qualitatively classify the stability of each steady-state equilibrium. We will identify the equation for $\dot{Y}$ and then illustrate steps (c) and (d) for a specific example where $F(X) = rX(1 - X/K)$ and $\pi(X,Y) = \ln[Y] + \beta\ln[X]$.

Taking a time derivative of the maximal condition will imply

$$\dot{\mu} = \pi_{YX}\dot{X} + \pi_{YY}\dot{Y} \tag{19}$$

where we adopt the more convenient notation $\pi_Y = \partial\pi(\cdot)/\partial Y$, $\pi_{YX} = \partial^2\pi(\cdot)/\partial Y\partial X$, *ete.* Noting that $\mu = \pi_Y$ and $\dot{X} = F(X) - Y$ we can substitute μ and $\dot{\mu}$ into equation (16) and solve for $\dot{Y}$ yielding

$$\dot{Y} = \frac{-\pi_{YX}[F(X) - Y] - \pi_X + [\delta - F'(X)]\pi_Y}{\pi_{YY}} \tag{20}$$

The dynamical system of interest becomes

$$\begin{aligned} \dot{X} &= F(X) - Y \\ \dot{Y} &= \frac{-\pi_{YX}[F(X) - Y] - \pi_X + [\delta - F'(X)]\pi_Y}{\pi_{YY}} \end{aligned} \tag{D1}$$

The isoclines for this system are the curves where $\dot{X} = 0$ and $\dot{Y} = 0$. Points where these two isoclines intersect are steady state equilibria and represent potentially sustainable points in X-Y space. The ability to reach a particular steady state will depend on its local stability.

To illustrate steps (c) and (d) above, suppose that the net benefit function takes the form $\pi = \ln[Y] + \beta\ln[X]$, where $\ln[\cdot]$ is the natural log operator and $\beta > 0$ measures the relative weight placed on the flow of nonconsumptive benefits from the resource stock. Note $\pi_Y = 1/Y$, $\pi_X = \beta/X$, $\pi_{YX} = 0$, and $\pi_{YY} = -1/Y^2$. Further, suppose $F(X) = rX(1 - X/K)$. This form for F(X) is known as the *logistic growth function.* Note $F'(X) = r(1 - 2X/K)$. The $\dot{Y} = 0$ isocline implies

$$F'(X) + \pi_X/\pi_Y = \delta \tag{21}$$

This is a famous equilibrium condition for a renewable resource. It can be given a capital-theoretic interpretation. On the left-hand-side (LHS) we have two terms, the marginal net rate of growth, $F'(X)$, and what is called the *marginal stock effect,* π_X/π_Y. The sum of these two terms represents the resource's own rate of return. On the RHS we have the discount rate, δ. Thus, at the optimal steady state we want the resource stock, X, and level of harvest, Y, to cause the resource's own rate of return to equal the rate of discount, which in a competitive economy, would equal the rate of return on other investments.

The $\dot{X} = 0$ isocline implies Y = F(X), which must hold for any steady state, and, as noted in Section 1, simply requires that harvest equals net growth. Given our functional forms for π(X,Y), F(X), and their derivatives, we can show that $\dot{Y} = 0$ implies

$$Y = \Phi(X) = \frac{X}{\beta}\left[\frac{2rX}{K} - (r - \delta)\right] \tag{22}$$

and $\dot{X} = 0$ implies

$$Y = rX(1 - X/K) \tag{23}$$

A plot of these two isoclines, in the X-Y phase plane, is shown in Figure 2 for r=1, K=1, β=4, and δ=0.02.

(Figure 2. about here)

There are two steady-states, corresponding to the origin, (0,0) and (X^*, Y^*), where the intersection of Φ(X) and F(X) for X > 0 occurs at

$$X = \frac{K[r\beta + (r - \delta)]}{(2r + r\beta)} \tag{24}$$

For r=1, K=1, β=4, and δ=0.02, equation (24) yields $X^* = 0.83$ and equation (23) implies $Y^* = 0.1411$. If X(0) > 0, which steady state is optimal? Is the optimal steady state stable? If stable, how would we approach X^* from X(0)? To answer these questions we need to determine the directionals, or vector field, for each isosector in Figure 2.

We know that along Y = F(X), that $\dot{X} = 0$. For a point above or below F(X), $\dot{X}$ will be positive or negative. We can determine the direction of change by noting that the sign of $\partial \dot{X} / \partial Y < 0$. This implies that if we increase Y, so we move to a point above F(X), that $\dot{X} < 0$, and, conversely, if we decrease Y, so that we move to a point below F(X), that $\dot{X} > 0$. Similarly, we know along Y = Φ(X) that $\dot{Y} = 0$. For a point to the left or right of Φ(X), $\dot{Y}$ will be positive or negative. By noting that $\partial \dot{Y} / \partial X < 0$ we know that if we move to a point to the left of Φ(X) (thus decreasing X) that $\dot{Y} > 0$, and, conversely, if we move to a point to the right of Φ(X) (thus increasing X) that $\dot{Y} < 0$. These signs for $\dot{X}$ and $\dot{Y}$ will imply the directionals in the four isosectors of Figure 2 and they imply that (X^*,Y^*) is *saddle-point stable.* It can also be shown that the origin is locally unstable. If $X(0) > 0$ we could approximate the optimal feed-back control policy, $Y^* = \varphi(X)$, that would allow us to asymptotically approach (X^*,Y^*). A simple way to do this, within a discrete-time model, will be illustrated in the next subsection.

2.2 A Discrete-Time, Deterministic Model

It is useful to present the discrete-time, deterministic model for two reasons. First, the discrete-time model can be programmed on a spreadsheet and finite-horizon problems can be solved using the nonlinear programming routines that come with most modern spreadsheets. Second, the discrete-time deterministic model leads naturally to the stochastic model, just as equation (4) lead naturally to equation (5). In discrete time our renewable resource management problem seeks to

$$\begin{aligned}&\text{Maximize} \quad \sum_{t=0}^{\infty} \rho^t \pi(X_t, Y_t) \\ &\text{Subject to} \quad X_{t+1} - X_t = F(X_t) - Y_t, X_0 \text{ given}\end{aligned} \tag{P2}$$

Recall that $\rho = 1/(1 + \delta)$ is our discount factor and δ is the periodic discount rate. This problem may be solved using the Method of Lagrange Multipliers. The Lagrangian expression might be formulated at least two ways, depending on the desired interpretation of the Lagrange multipliers. We will formulate the Lagrangian as

$$L = \sum_{t=0}^{\infty} \rho^t \{\pi(X_t, Y_t) + \rho\lambda_{t+1}[X_t + F(X_t) - Y_t - X_{t+1}]\} \tag{25}$$

where λ_{t+1} is the (current-value) shadow price of the resource in period t+1. The term $\lambda_{t+1}[X_t + F(X_t) - Y_t - X_{t+1}]$ can be thought of as a capital gain (or loss) term in period t+1. To make it comparable to the net benefit flow in period t, it is discounted once, and added to $\pi(X_t,Y_t)$. The first-order necessary conditions, assuming Y_t, X_t, and λ_{t+1} will be positive for all t, require

$$\partial L / \partial Y_t = \rho^t \{\partial \pi(\cdot) / \partial Y_t - \rho\lambda_{t+1}\} = 0 \tag{26}$$

$$\partial L / \partial X_t = \rho^t \{\partial \pi(\cdot) / \partial X_t + \rho\lambda_{t+1}[1 + F'(X_t)]\} - \rho^t \lambda_t = 0 \tag{27}$$

$$\partial L / \partial(\rho\lambda_{t+1}) = \rho^t \{X_t + F(X_t) - Y_t - X_{t+1}\} = 0 \tag{28}$$

In equation (27), the term $-\rho^t\lambda_t$ arises because in the (t-1) term of the Lagrangian, where most variables are subscripted (t-1), the last term is X_t, with a derivative yielding the last term in (27). In equation (28) we take a derivative with respect to the discounted current-value shadow price. Some authors set up the Lagrangian with the term $\eta_t[X_t + F(X_t) - Y_t - X_{t+1}]$ in which case $\eta_t = \rho\lambda_{t+1}$ is a discounted multiplier. We prefer current-value multipliers (they represent what you'd be willing to pay for an additional unit of the resource deliverable in the current period) to discounted multipliers (they represent what you would be willing to pay today for delivery of an additional unit of the resource in the next period).

In addition to the above first-order conditions, we again have the initial condition on the resource stock, X_0, and the transversality condition, which in discrete time becomes $\lim \rho^t \lambda_t X_t \rightarrow 0$ as

$t \rightarrow \infty$. As in the continuous-time model, we might inquire into the existence, uniqueness, and stability of steady-state equilibria. In steady state, equations (26) - (28) imply

$$\pi_Y = \rho\lambda \tag{29}$$

$$\rho\lambda[1+F'(X)-(1+\delta)] = -\pi_X \tag{30}$$

$$Y = F(X) \tag{31}$$

Doing a bit more algebra on equation (30) yields $\rho\lambda[\delta - F'(X)] = \pi_X$. Substituting (29) yields

$$F'(X) + \pi_X/\pi_Y = \delta \tag{32}$$

which is identical to equation (21), the steady-state, capital-theoretic, equation from the continuous-time model. Thus, equations (31) and (32) define the steady-state optima for (P1) and (P2). Because (P1) presumes continuous- and (P2) discrete-time, there may be a difference in the approach dynamics and stability of (X^*,Y^*). The discrete-time difference equations in the first-order conditions of (P2) have the potential for overshoot and can result in more unusual dynamics, including *deterministic chaos.*

Problem (P2) does have the advantage of being "numerically friendly," in that it is easily coded on a spreadsheet and is potentially solvable using nonlinear programming. Further, it may be possible to approximate the optimal approach from X_0 to X^* for an infinite-horizon problem. This will require finding a *final function* that approximates the present value of reaching the steady-state optimum. Let's return to the functional forms adopted at the end of the preceding section to illustrate steps (c) and (d) in the qualitative analysis of dynamical system (D1). Specifically, we assumed $\pi_t = \ln[Y_t] + \beta\ln[X_t]$ and $F(X_t) = rX_t(1 - X_t/K)$. Suppose at some future date, t=T, we decide that we will sustainably harvest the stock, X_T, forever. What is the present value of such a policy? Well, it depends on the stock in period T, which depends on the initial stock X_0 and our harvest policy, Y_t, for t=0,1,2,...,T-1. Given X_T, we know our sustainable level of harvest is $Y = rX_T(1 - X_T/K)$. In period T the present value of the policy over the horizon t=T,T+1,...,∞ converges according to

$$PV_T = \{\ln[rX_T(1-X_T/K)] + \beta\ln[X_T]\}\sum_{t=0}^{\infty}\rho^t = \{\ln[rX_T(1-X_T/K)] + \beta\ln[X_T]\}\left[\frac{(1+\delta)}{\delta}\right] \tag{33}$$

Discounting this final function back to t=0 gives

$$PV_{T,0} = \rho^{T-1}\{\ln[rX_T(1-X_T/K)] + \beta\ln[X_T]\}/\delta \tag{34}$$

We then wish to find the harvest schedule Y_t, t=0,1,2,...,T-1 which will

$$\begin{aligned}&\text{Maximize} \quad PV = \sum_{t=0}^{T-1}\rho^t\{\ln[Y_t] + \beta\ln[X_t]\} + \rho^{T-1}\{\ln[rX_T(1-X_T/K)] + \beta\ln[X_T]\}/\delta \\ &\text{Subject to} \quad X_{t+1} - X_t = rX_t(1-X_t/K) - Y_t,\ X_0 \text{ given}\end{aligned} \tag{P3}$$

This problem is coded on the spreadsheet shown in Table 1.

(Table 1. about here)

The parameter values r=1, K=1, β=4, and δ=0.02 are entered in cells B4 through B7. These are the same parameter values that gave rise to the isoclines drawn in Figure 2 with $X^* = 0.83$ and $Y^* = 0.1411$ being saddle-point stable. In cells A10 though A30 we enter the period index, where T=20 and we wish to determine the optimal harvest rate for t=0,1,2,...,19. The terminal stock is X_{20}, which must be maintained forever by harvesting $F(X_{20}) = rX_{20}(1 - X_{20}/K)$. In cells C10 through C29 we introduce a "guess," or candidate schedule for Y_t^*, setting $Y_t = 0.1$ for t=0,1,2,...,19. In cell B10 we enter the initial condition $X_0 = 0.2$. In cell B11 we program

=B10+$B$4*B10*(1-B10/$B$5)-C10 (35)

which is the spreadsheet equivalent of $X_1 = X_0 + rX_0(1 - X_0/K) - Y_0$. We then fill down from B11 through B30. By placing $ before the column letter and row number for parameters, they remain fixed when we fill down, whereas the variables X_t and Y_t, without $, will automatically increment so that in cell B30 the fill down from B11 yields

0.88729833=B29+$B$4*B29*(1-B29/$B$5)-C29 (36)

In cell D10 we program the spreadsheet version of $p_0 = \ln[Y_0] + \beta\ln[X_0]$ which takes the form

=((1/(1+$B$7))^A10)*(LN(C10)+$B$6*LN(B10)) (37)

We fill down through D29. In cell D30 we program the spreadsheet equivalent of equation (34) which takes the form

=((1/(1+$B$7))^$A$29)*(LN($B$4*$B$30*(1-$B$30/$B$5))+$B$6*LN($B$30))/$B$7 (38)

This is the present value of sustainably harvesting X_{20} for t=20,21,...,∞. In cell D32 we sum the discounted net benefit flows for t=0,1,2,...,19 and the final function by entering

=SUM($D$10:$D$30) (39)

This corresponds to the objective function in (P3). We now call up Excel's Solver, a nonlinear programming routine. In Solver's Parameter Window the Set Cell should be $D$32, which we wish to maximize by allowing Solver to change our guess for Y_t^* by changing the values in cells $B$10:$B$29. We impose the non-negativity constraint $B$10:$B$29 >= 0, which turns out to be non binding. Clicking on the "Solve" button, Solver will go through 19 trial iterations before converging to the optimal solution shown in Table 2.

(Table 2. about here)

The candidate solution has been changed so that the harvest rate is reduced to $Y_0^* = 0.02728809$, and allowed to slowly increase to $Y_{19}^* = 0.14114552 \approx Y^*$, while the stock slowly grows from $X_0 = 0.2$ to $X_{20}^* = 0.8300061 \approx X^*$. Solver has thus converged to our previously determined steady-state optimum (X^*,Y^*). The optimal approach is plotted in Figure 3, where the points (X_6^*, Y_6^*) through (X_{19}^*, Y_{19}^*) "overlap" because of their proximity to (X^*,Y^*).

(Figure 3. about here)

2.3 A Discrete-Time, Stochastic Model

The resource management problem, based on the stochastic growth process described by equation (5), might be stated as

$$\text{Maximize} \quad E_0\{\sum_{t=0}^{T} \rho^t \pi(X_t Y_t)\}$$
$$\text{Subject to} \quad X_{t+1} = G(X_t, Y_t; z_{t+1}),\ f(z_{t+1}),\ X_0 \ \text{given} \qquad \text{(P4)}$$

In contrast to problems (P1) and (P2), we begin with a finite-horizon problem where $t = 0,1,2,...,T$. We will then allow $T \to \infty$ and solve for the stationary feedback policy, $Y_t^* = \phi(X_t)$.

Suppose (a) that $\pi(\cdot)$ is concave in X_t and Y_t in all periods, (b) that it is infinitely costly to harvest the entire stock in any period, and (c) that $Y_t \geq 0$. These assumptions will imply $X_t > Y_t^* \geq 0$. In the terminal period, t=T, where it is assumed that the terminal stock can be observed with certainty, net benefit is simply $\pi(X_T,Y_T)$. Maximization of $\pi(X_T,Y_T)$ with respect to Y_T is a simple static optimization problem. Suppose in t=T that $Y_T^* > 0$ so that the first-order condition is simply $\partial\pi(X_T,Y_T)/\partial Y_T = 0$ and by the implicit function theorem there exits a feedback policy $Y_T^* = \phi_T(X_T)$. Knowing the feedback policy in the terminal period, the optimized value function becomes $V_T(X_T) = \pi[X_T, \phi_T(X_T)]$.

The value function in period t=T-1 may be written as

$$V_{T-1}(X_{T-1}) = \text{Max}[\pi(X_{T-1}, Y_{T-1}) + \rho E_{T-1}\{V_T(G(X_{T-1}, Y_{T-1}; z_T))\}] \qquad (40)$$

where $X_T = G(X_{T-1},Y_{T-1};z_T)$ has been substituted into $V_T(X_T)$, and the expectation operator, $E_{T-1}\{\cdot\}$, is over the induced random variable X_T, given X_{T-1}, Y_{T-1}, and the density $f(z_T)$. Assume that the expectation operation is well defined in t=T-1 and in each earlier period as we proceed by backward induction toward $E_0\{\cdot\}$. With this assumption, and assuming $Y_{T-1}^* > 0$, the maximal condition is again a partial derivative which requires

$$\partial\pi(\bullet)/\partial Y_{T-1} + \rho\partial E_{T-1}\{V_T(G(X_{T-1}, Y_{T-1}; z_T))\}/\partial Y_{T-1} = 0 \qquad (41)$$

This is one equation in Y_{T-1} and X_{T-1}, since the expectation operator will "integrate out z_T." The implicit function theorem will imply a feedback policy where $Y_{T-1}^* = \phi_{T-1}(X_{T-1})$. Substituting this feedback policy into (40) yields the desired value function, which simply depends on X_{T-1}, but which presumes optimal behavior in the form of $Y_{T-1}^* = \phi_{T-1}(X_{T-1})$ and $Y_T^* = \phi_T(X_T)$.

In period t=T-2 the value function may be written as

$$V_{T-2}(X_{T-2}) = \text{Max}[\pi(X_{T-2}, Y_{T-2}) + \rho E_{T-2}\{V_{T-1}(G(X_{T-2}, Y_{T-2}; z_{T-1}))\}] \qquad (42)$$

The maximal condition requires

$$\partial\pi(\bullet)\,/\,\partial Y_{T-2} + \rho\partial E_{T-2}\{V_{T-1}(G(X_{T-2},Y_{T-2};z_{T-1}))\}/\partial Y_{T-2} = 0 \tag{43}$$

The maximal condition will imply the feedback policy $Y^*_{T-2} = \phi_{T-2}(X_{T-2})$ and substitution will yield $V_{T-2}(X_{T-2})$ for the next backward step. The generic forms for the value function, maximal condition, and optimal feedback policy were given as equations (9) - (11) in Section 1.2. It can be shown that as $T \to \infty$, $\phi_t(X_t) \to \phi(X_t)$; that is, the optimal, period-specific, feedback policies collapse to a single stationary policy which holds in every period.

It would be nice to illustrate the use of dynamic programming to solve (P4) when $F(X_t) = r_{t+1}X_t(1 - X_t/K)$, $\pi(X_t,Y_t) = \ln[Y_t] + \beta\ln[X_t]$, and $f(r_{t+1})$ is a probability density for the random intrinsic growth rate. Unfortunately, analytic solutions for $\phi_t(X_t)$ and $V_t(X_t)$ are not possible. In Section 3 we will present a more tractable special case.

2.4 A Continuous-Time, Stochastic Model

The model of this subsection is based on a stochastic differential equation, equation (6), from Subsection 1.1. Consider the problem

$$\begin{aligned}&\text{Maximize} \quad E_0\left\{\int_0^\infty \pi(X,Y)e^{-\delta t}dt\right\}\\ &\text{Subject to} \quad dX = [F(X) - Y]dt + \sigma(X)dz,\ X(0)\ \text{given}\end{aligned} \tag{P5}$$

As in the discrete-time problem (P4), we will make use of dynamic programming in an attempt to solve this problem. For this infinite-horizon, continuous-time problem we may write the value function at instant t as

$$V(X) = \text{Max}\left[E_t\left\{\int_t^\infty \pi(X(\tau),Y(\tau))e^{-\delta(\tau-t)}d\tau\right\}\right] \tag{44}$$

where the maximization is with respect to $Y(\tau)$ for $\infty > \tau \geq t$. The equation of optimality, often called the Hamilton-Jacobi-Bellman (H-J-B) equation, requires

$$\delta V(X) = \text{Max}[\pi(X,Y) + (1/dt)E_t\{dV\}] \tag{45}$$

The expression to be maximized can again be interpreted as the sum of the instantaneous net benefit flow, $\pi(X,Y)$, plus an expected capital gain term, $(1/dt)E_t\{dV\}$.V(X) is a function of a stochastic (Itô) variable, and taking the differential, dV, requires the use of stochastic calculus. The Itô Calculus has been argued to be the more compelling calculus for economic applications and Itô's Lemma can be used to show that

$$(1/dt)E_t\{dV\} = [F(X) - Y]V'(X) + (\sigma^2(X)/2)V''(X) \tag{46}$$

which when substituted into the H-J-B equation yields

$$\delta V(X) = \text{Max}[\pi(X,Y) + (F(X) - Y)V'(X) + (\sigma^2(X)/2)V''(X)] \tag{47}$$

Assuming Y > 0, the maximal condition is a partial derivative requiring

$$\pi_Y - V'(X) = 0 \tag{48}$$

Note the similarity between equation (15) and equation (48). V´(X) is playing the role of μ. V´(X) is the expected present value of having one more unit of X *in situ*, at instant t, and it must be equated to π_Y, the net marginal value of harvesting one more unit today. If we knew the form of V(X), we could take the first derivative and know V´(X). Then equation (48) would be one equation in X and Y, and because it is an optimality condition, we could solve it for $Y^* = \varphi(X)$, the optimal, stationary harvest policy. Suppose this is possible. Then we could rewrite (47) as

$$\delta V(X) = \pi(X,\varphi(X)) + [F(X) - \varphi(X)]V'(X) + (\sigma^2(X)/2)V''(X) \tag{49}$$

Equation (49) is a nonlinear, nonhomogeneous, second-order, ordinary differential equation. Its solution is the unknown function V(X) which has derivatives V´(X) and V´´(X) that satisfy the equation. In general, an analytic solution for V(X) is unlikely. There are special cases where it is possible to determine explicit forms for V(X) and $Y^* = \varphi(X)$ and to determine the *steady-state distributions* for X and Y. It is important to remember that the standard deviation rate, $\sigma(X)$, will cause the harvested stock to fluctuate, and that the fluctuating stock, via $Y^* = \varphi(X)$, will cause harvest to fluctuate as well.

If the optimal harvest policy is followed, X evolves according to

$$dX = [F(X) - \varphi(X)]dt + \sigma(X)dz \tag{50}$$

It has been shown that, baring degeneration, the long-run distribution of the resource is given by

$$\omega_\infty(X) = \frac{m}{\sigma^2(X)} \exp\left[2\int^X \frac{[F(\nu) - \varphi(\nu)]}{\sigma^2(\nu)} d\nu\right] \tag{51}$$

where m is a constant of integration chosen so that $\int_0^\infty \omega_\infty(X)dX = 1$. The expected steady-state stock is given as

$$\bar{X}_\infty = \int_0^\infty X\omega_\infty(X)dX \tag{52}$$

As with the discrete-time stochastic model, the derivation of the value function, V(X), and the stationary feedback harvest policy, $\varphi(X)$, will be illustrated for a special case in the next section.

3. Special Cases: Fisheries, Forestry, and Groundwater

3.1 Fisheries Management: The Linear Model

One of the first production functions used in modeling the economics of a commercial fishery assumed that $Y = qXE$, where Y is the rate of harvest, X is the fish stock, E is fishing "effort," and $q > 0$ is a "catchability coefficient." Fishing effort should be interpreted as some aggregate of inputs, including a vessel (capital), crew (labor), and other factors needed for fishing. If the unit cost of effort is given by the constant $\omega > 0$, then the cost of fishing at instant t is $C = \omega E$. If we solve the production function for $E = Y/(qX)$, cost can be written as a function of harvest and stock size so that $C = \omega Y/(qX)$. Finally, if the price per unit of harvest (on the dock) is $p > 0$, then the net revenue at instant t may be written as

$$\pi = pY - \omega Y/(qX) = [p - c/X]Y \tag{53}$$

where $c = \omega/q > 0$. Suppose the net growth function is logistic so that $F(X) = rX(1 - X/K)$. The current-value Hamiltonian, written in general form as equation (14), becomes

$$H = (p - c/X - \mu)Y + \mu F(X) \tag{54}$$

and is linear in Y. The term $\sigma = p - c/X - \mu$ is sometimes called *the switching function.* When maximizing H (and thus the present value of net revenues), if $\sigma > 0$, you would find it optimal to make Y as large as possible. Conversely if $\sigma < 0$, you would want to make Y as small as possible. Suppose that $Y_{MAX} \geq Y \geq 0$, and that $Y_{MAX} > F(X_{msy})$. Then if $\sigma > 0$, $Y^* = Y_{MAX}$, and if $\sigma < 0$, $Y^* = 0$. There will exist a unique, stock level, $X^* > 0$, which when reached is maintained by setting $Y^* = rX^*(1 - X^*/K)$. This is the optimal, steady-state, stock, and because $Y^* = Y_{MAX}$ when $\sigma > 0$ (because $X > X^*$), and $Y^* = 0$ when $\sigma < 0$ (because $X < X^*$), the approach to X^* from some $X(0) \neq X^*$ is said to "most rapid." In words, when the fish stock is above its optimum ($X > X^*$), you harvest at the maximum rate possible, and when the stock is below its optimum ($X < X^*$), you declare a moratorium on fishing. With π given in equation (53) and $F(X) = rX(1 - X/K)$, the optimal steady-state stock is implied by equation (21) and has the analytic form

$$X^* = \frac{K}{4}\left[\left(\frac{c}{pK} + 1 - \frac{\delta}{r}\right) + \sqrt{\left(\frac{c}{pK} + 1 - \frac{\delta}{r}\right)^2 + \frac{8c\delta}{pKr}}\right] \tag{55}$$

The optimal, steady-state stock is the positive root of a quadratic. It depends on five bioeconomic parameters: c, δ, K, p, and r. The optimal stock may be greater or less than X_{msy}, depending on the particular set of bioeconomic parameters.

Consider the linear model when $c=600$, $\delta=0.08$, $K=1{,}000$, $p=2$ and $r=0.4$. Substituting these parameter values into (55) yields $X^* = 600 > X_{msy} = 500$. Suppose that prior to management, the fishery had been harvested under "open access" conditions and that net revenue, $\pi = (p - c/X)Y$, had been driven to zero. The open-access, equilibrium stock is $X_\infty = c/p = 300$. Suppose fishery managers wish to optimally increase the stock from $X(0) = X_\infty = c/p = 300$ to $X^* = 600$. We know that the most rapid approach path (MRAP) is optimal, and with $X(0) < X^*$ we know that $Y^* = 0$ until the stock grows to X^*. How long will the moratorium be?

During the moratorium we know that the stock will grow according to $\dot{X} = rX(1 - X/K)$. This differential equation has an analytic solution given by

$$X(t) = \frac{K}{[1 + ae^{-rt}]} \tag{56}$$

where a = [K - X(0)]/X(0). If X(0) = X_∞ = c/p, then a = [pK - c]/c. The moratorium has to last until X(t) = X^*, or

$$X^* = \frac{cK}{[c + (pK - c)e^{-rt}]} \tag{57}$$

Carefully solving for t will show that the moratorium must last until

$$t^* = (1/r)\ln\left[\frac{X^*(pK - c)}{c(K - X^*)}\right] \tag{58}$$

For X^* = 600, and the previous parameter values of r=0.4, p=2, K=1,000, and c=600, we calculate t^* = 3.132. The entire solution to the linear fishery problem is shown in Figure 4.

(Figure 4. about here)

To summarize, from X(0) = X_∞ = c/p = 300 it is optimal to impose a moratorium (Y^* = 0) until t^* = 3.132, at which time $X(t^*) = X^*$ = 600, and fishing can commence with a steady-state optimal harvest of $Y^* = rX^*(1 - X^*/K)$ = 96.

3.2 Forest Management: The Faustmann Model

In contrast to commercial fishing, where inputs and outputs might be continuous over time, forestry, or at least rotational forestry, is often described as a point-input, point-output process. Seedlings might be planted (the input), allowed to grow, and at a certain age the trees are cut (yielding an output). Instead of a stock-dependent, net growth function, F(X), the seedlings, planted at t = 0, are assumed to grow so that at t > 0, their merchantable volume is Q(t). Suppose at t = T it is optimal to cut the stand of trees and to replant the parcel of land. At t = T, the volume of timber is Q(T). Let the net per unit price for timber be p > 0 and the cost of replanting be c > 0. At t = T, revenue, net of replanting cost, is given by [pQ(T) - c]. If the volume function, Q(t), p, c, and the discount rate, δ, are unchanging, the optimal length of time to allow the second stand to grow will also be T years and the second stand will be cut at t = 2T. Suppose this is the case for all future stands so that the parcel is cut and replanted at T, 2T, 3T, and so on *ad infinitum.* Finally, assume that the initial batch of seedlings has been paid for and planted. What is the discounted net revenue of a rotation of length T? This present value may be written as

$$\pi(T) = [pQ(T) - c]e^{-\delta T}\{1 + e^{-\delta T} + (e^{-\delta T})^2 + (e^{-\delta T})^3 + \cdots\} \tag{59}$$

With $\delta > 0$, $1 > e^{-\delta T} > 0$ and the series in $\{\cdot\}$ will converge to $1/(1 - e^{-\delta T})$. $\pi(T)$ may be written as

$$\pi(T) = \frac{[pQ(T) - c]e^{-\delta T}}{(1 - e^{-\delta T})} = \frac{[pQ(T) - c]}{(e^{\delta T} - 1)} \tag{60}$$

The rotation that maximizes $\pi(T)$ is called the Faustmann rotation. Setting $d\pi(T)/dT = 0$, and doing some careful algebra, you can show that the following equation is implied

$$pQ'(T) = \delta[pQ(T) - c] + \delta\pi(T) \tag{61}$$

Equation (61) has a nice economic interpretation. On the LHS, $pQ'(T)$ is the marginal value of letting the current stand grow a bit longer. On the RHS there are two terms. The first, $\delta[pQ(T) - c]$, is the interest payment foregone on the current stand, by letting it grow a bit longer. The second term, $\delta\pi(T)$, can be shown to equal the marginal cost of delaying all future stands. Thus, the RHS is the marginal cost of delaying the cutting of the current stand. The Faustmann rotation precisely balances the marginal value of delay (allowing the current stand to grow a bit longer) with the marginal cost of delay (the foregone interest payment on the net revenue from the current stand and delaying the harvest of all future stands).

Consider the case when $Q(t) = e^{a - b/t}$, $b > a > 0$. This functional form seems to give a good fit to volume and age data for commercially grown trees. As $t \to \infty$, $Q(t) \to e^{a}$. For Douglas fir, on moderately good land in the Pacific Northwest, it has been estimated that a = 13 and b = 196, where Q(t) measures the volume of timber in board feet per acre. A plot of this volume function is shown in Figure 5.

(Figure 5. about here)

If p = \$0.65/board foot, c = \$180/acre (replanting cost) and $\delta = 0.05$, the Faustmann rotation is $T^* = 61.62$ years and $\pi(T^*) = \$566.37$/acre. Our equation for $\pi(T)$, equation (60), assumed that the seedlings for the first rotation had been paid for and planted. Thus, $\pi(T^*)$ gives us the optimized value of *recently replanted land* if it is devoted to forestry. After cutting, it must be the case that $\pi(T^*) - c > 0$ to make replanting worthwhile. In this example $\pi(T^*) - c = \$566.37 - \$180 = \$386.37$, so replanting and continued forestry has a positive present net value under the Faustmann rotation.

3.3 Groundwater

In arid and semi-arid parts of the world, groundwater is used for human consumption, watering livestock, and irrigation. The water is pumped from an aquifer, with the cost of pumping affected by the height of the "water table" relative to the surface. The greater the distance between water table and surface, the greater the "lift," and the higher the unit cost. Let X denote the stock of groundwater and we will assume that the function $c(X) = c/X$ defines the unit cost of pumping, where $c > 0$. At the surface, assume that each unit of water would fetch a constant price, $p > 0$. This gives rise to the net revenue $\pi = (p - c/X)Y$ as in equation (53) from the linear fishery, only now, Y is the number of units of water pumped from the aquifer at instant t. Suppose that the capacity of pipes and pump is such that $Y_{MAX} \geq Y \geq 0$.

Finally, assume that the aquifer is replenished (or recharged) at the known constant rate $R > 0$ and that $Y_{MAX} > R$. The change in the groundwater stock is given by

$$\dot{X} = R - Y \tag{62}$$

The groundwater optimization problem becomes

$$\text{Maximize} \quad \int_0^{\infty} (p - c/X)Ye^{-\delta t}dt$$

$$\text{Subject to} \quad \dot{X} = R - Y \tag{P6}$$

$$Y_{MAX} \geq Y \geq 0,\ Y_{MAX} > R > 0,\ X(0) > 0 \text{ given}$$

where $\delta > 0$ is the discount rate. The current-value Hamiltonian for (P6) is given by

$$H = (p - c/X)Y + \mu(R - Y) = (p - c/X - \mu)Y + \mu R \tag{63}$$

which is linear in Y so we know that the most rapid approach path (MRAP) to the optimal groundwater stock, X^*, will be optimal. Aside from the switching function, $\sigma = p - c/X - \mu$, the first-order conditions include

$$\dot{\mu} - \delta\mu = -\partial H/\partial X = -cY/X^2 \tag{64}$$

$$\dot{X} = \partial H/\partial \mu = R - Y \tag{65}$$

In steady state $\dot{X} = \dot{\mu} = 0$, $\sigma = 0$, and $\mu = p - c/X$. Equation (64) implies

$$Y = \Phi(X) = (\delta/c)X(pX - c) \tag{66}$$

while equation (65) implies

$$Y^* = R \tag{67}$$

The intersection of $\Phi(X)$ with $Y^* = R$ will define the steady-state, optimal groundwater stock, X^*. Equations (66) and (67) are shown in Figure 6 for the parameter values δ=0.04, c=400, p=4.8, and R=100. In this model X^* has an explicit solution as given by

$$X^* = \frac{c/p + \sqrt{(c/p)^2 + 4cR/(\delta p)}}{2} \tag{68}$$

(Figure 6. about here)

For the above parameter values $X^* = 500$ and $Y^* = R = 100$. Suppose X(0) = 1,000 and $Y_{MAX} = 200$. We know the MRAP from X(0) to X^* is optimal and with $X(0) > X^*$ that $Y^* = Y_{MAX}$ until $X(t) = X^*$. Along the MRAP, $\dot{X} = R - Y_{MAX}$. This differential equation has the solution $X(t) = X(0) - (Y_{MAX} - R)t$ and implies that we would reach X^* from $X(0) > X^*$ at

$$t^* = \frac{[X(0) - X^*]}{(Y_{MAX} - R)} \tag{69}$$

For the above values of X(0), X^*, Y_{MAX} and R, $t^* = 5$ and we have the complete solution (steady state and optimal approach) to our groundwater management problem.

3.4 The Stochastic Forest

It is often the case that there is uncertainty in the future rate of growth for a resource and in its future price. Consider a parcel of land recently planted with seedlings of a commercial species of tree. Future rates of growth in the volume of timber may depend on future precipitation, insect populations, or other random events. The future price of timber may depend on interest rates, housing starts, and the price of substitute building materials. In this subsection we will determine the optimal time to cut a single stand of even-aged trees when the future growth of timber and its net unit price (or stumpage price) are stochastic processes.

Recall from our discussion of the Faustmann model, in Subsection 3.2, that Q = Q(t) was the volume of merchantable timber at instant t > 0 assuming that the parcel of land had been planted with seedlings at t = 0. Define q = ln(Q), where ln(·) is the natural log operator. Then

$$dq = [Q'(t)/Q(t)]dt = g(t)dt \tag{70}$$

We will define g(t) as the expected relative rate of growth. For example, in the case of our exponential volume function, where $Q(t) = e^{a - b/t}$, $g(t) = b/t^2$.

In our stochastic model we assume that the random factors affecting growth can be modeled by adding the term $\sigma_q dz_q$ to the expected relative rate of growth, where $\sigma_q > 0$ is a standard deviation rate, $dz_q = \varepsilon(t)\sqrt{dt}$, and $\varepsilon(t) \sim N(0,1)$. The log of volume then evolves stochastically according to

$$dq = g(t)dt + \sigma_q dz_q \tag{71}$$

Define p = ln(P) where P = P(t) is the net per unit price for timber at instant t. Assume that the log of price evolves according to

$$dp = \mu dt + \sigma_p dz_p \tag{72}$$

where μ is the expected relative rate of growth in price, and may be positive, zero, or negative, $\sigma_p > 0$ is the standard deviation rate for the log of price, $dz_p = \varepsilon(t)\sqrt{dt}$, and $\varepsilon(t) \sim N(0,1)$. We can allow for correlation in the processes for q and p by specifying that $E\{dz_q, dz_p\} = \rho dt$, where ρ is a correlation coefficient, $1 \geq \rho \geq -1$.

When is it optimal to cut this single stand of trees and sell the timber? Presumably there will be an interval of time where it is optimal to let the trees stochastically grow. During this interval, called the continuation region, the stand of trees will have an option value, based on the assumption that the stand will be cut at the optimal time, T^*, in the future. To be indifferent between holding the stand (continued ownership) or selling the stand (and thus the option to optimally harvest at T^*), the interest payment on the value function, $\delta V(p,q)$ must equal the expected capital gain from continued ownership. This implies

$$\delta V(p,q) = (1/dt)E_t\{dV(p,q)\} \tag{73}$$

Using Itô's Lemma to take the stochastic differential, taking the expected value and then multiplying through by (1/dt) will imply

$$\delta V(p,q) = g(t)V_q(p,q) + \mu V_p(p,q) + (\sigma_q^2/2)V_{qq}(p,q) + (\sigma_p^2/2)V_{pp}(p,q) + \rho\sigma_q\sigma_p V_{qp}(p,q) \tag{74}$$

The value function satisfying this partial differential equation and the boundary condition that $V(p(T),q(T)) = P(T)Q(T)$ is $V(p,q) = e^{p+q}$. Note that all the partial derivatives also equal e^{p+q}, so

$$\delta e^{p+q} = [g(t) + \mu + \sigma_q^2/2 + \sigma_p^2/2 + \rho\sigma_q\sigma_p]e^{p+q} \tag{75}$$

At the optimal time to harvest equation (75) implies

$$g(T^*) = \delta - (\mu + \sigma_q^2/2 + \sigma_p^2/2 + \rho\sigma_q\sigma_p) = \Delta \tag{76}$$

If $g(0) > \Delta > g(\infty) \geq 0$ and if g(t) monotonically decreases, there will exist a unique T^*. Returning to our exponential volume function, $Q(t) = e^{a - b/t}$, which implied that $g(t) = b/t^2$, we can show that $T^* = \sqrt{b/\Delta}$. For b=196, δ=0.05, μ=0, σ_q=0.1, σ_p=0.2, and ρ = - 0.9, we calculate that Δ = 0.043 and T^* = 67.51 years. If growth and price were deterministic (no uncertainty), it can be shown that the optimal age to harvest, assuming our exponential volume function, is given by $t^* = \sqrt{b/\delta}$ = 62.61 years. Thus, with no expected drift in price (μ=0), but variability in tree growth and the net price for timber, the optimal age to cut increases.

3.5 The Stochastic Fishery: Discrete Time

In this subsection we consider a discrete-time, stochastic model. The optimal harvest policy will be stationary and adaptive.

In many fishery models $S_t = X_t - Y_t$, the difference between stock size, X_t, and harvest, Y_t, is called *escapement.* The dynamics of the resource is given by the stochastic difference equation

$$X_{t+1} = z_{t+1}G(S_t) \tag{77}$$

where the z_{t+1} are i.i.d. random variables and $E\{z_{t+1}\} = 1$. Note that equation (77) is in the class of equations described earlier by equation (5).

Suppose that the fishery production function takes the form

$$Y_t = X_t(1 - e^{-qE_t}) \tag{78}$$

where E_t is fishing effort in period t and q > 0 is a catchability coefficient. Note if $E_t = 0$, $Y_t = 0$, and as $E_t \to \infty$, $Y_t \to X_t$. The cost of fishing in period t is assumed to depend only on the level of effort as given by the *cost equation* $C_t = cE_t$, where c > 0 is a cost coefficient. Solving the production function for effort one can show that

$$E_t = (1/q)\ln[X_t/(X_t - Y_t)] = (1/q)\ln[X_t/S_t] = (1/q)[\ln X_t - \ln S_t] \quad (79)$$

where ln[·] is the natural log operator. Substituting this last expression into the cost equation yields the *cost function*

$$C_t = (c/q)[\ln X_t - \ln S_t] \quad (80)$$

Net revenue in period t is given by

$$\pi_t = pY_t - (c/q)[\ln X_t - \ln S_t] = p(X_t - S_t) - (c/q)[\ln X_t - \ln S_t] \quad (81)$$

Now for a critical observation! If we define N(m) = pm - (c/q)ln(m) we can write net revenue as a *separable function* of X_t and S_t.

$$\pi_t = N(X_t) - N(S_t) \quad (82)$$

The problem of maximizing the expected value of discounted net revenue subject to stochastic recruitment may be stated as

$$\begin{aligned} &\text{Maximize} \quad E_0\left\{\sum_{t=0}^{T} \rho^t [N(X_t) - N(S_t)]\right\} \\ &\text{Subject to} \quad X_{t+1} = z_{t+1}G(S_t) \\ &\qquad X_0 \text{ given and } z_{t+1} \text{ i.i.d. with } E\{z_{t+1}\} = 1 \end{aligned} \quad \text{(P7)}$$

Momentarily we will let $T \to \infty$, but for now assume T is finite. Our solution to (P7) will employ stochastic dynamic programming and so we start in period t=T where the value function may be written as

$$V_T = \text{Max } [N(X_T) - N(S_T)] \quad (83)$$

Recall that value functions presume optimal behavior and in (83) we are assuming that the bracketed expression is being maximized by the choice of S_T. It is assumed that X_T is known and thus $N(X_T)$ is a constant. The first-order condition simply requires that $-\partial N(S_T)/\partial S_T = 0$. Given our definition of N(·) the first-order condition implies that $-p + c/(qS_T) = 0$ or $S_T = c/(pq)$. S_T is sometimes called *the open access escapement level* because it corresponds to zero net revenue or *rent dissipation* in the terminal period. The value function becomes $V_T = N(X_T) - N(c/(pq))$.

In period t=T-1, the value function is

$$\begin{aligned} V_{T-1} &= \text{Max } [N(X_{T-1}) - N(S_{T-1}) + \rho E_{T-1}\{V_T\}] \\ &= \text{Max } [\rho E_{T-1}\{N(z_T G(S_{T-1}))\} - N(S_{T-1}) + N(X_{T-1}) - \rho N(c/(pq))] \end{aligned} \quad (84)$$

In going from the first line to the second line in equation (84) we substitute $V_T = N(z_T G(S_{T-1})) - N(c/(pq))$. The last two terms on the second line of (84) are constants and the value of S_{T-1} which maximizes the bracketed expression also maximizes

$$W(S_{T-1}) = \rho E_{T-1}\{N(z_T G(S_{T-1}))\} - N(S_{T-1}) \tag{85}$$

If $W(S_{T-1})$ is quasi-concave, the first-order condition is also sufficient. It requires

$$W'(S_{T-1}) = \rho E_{T-1}\{N'(z_T G(S_{T-1})) z_T G'(S_{T-1})\} - N'(S_{T-1}) = 0 \tag{86}$$

With $G'(S_{T-1})$ independent of z_T equation (86) implies

$$G'(S_{T-1})\left[\frac{E_{T-1}\{z_T N'(z_T G(S_{T-1}))\}}{N'(S_{T-1})}\right] = (1+\delta) \tag{87}$$

Assuming that the expectation operation is well defined, equation (87) is one equation in one unknown and may be solved for S^*_{T-1}. We may then write V_{T-1} as

$$V_{T-1} = N(X_{T-1}) - N(S^*_{T-1}) + \rho E_{T-1}\{N(z_T G(S^*_{T-1}))\} - \rho N(c/(pq)) \tag{88}$$

In t=T-2 the value function is initially written as

$$\begin{aligned} V_{T-2} &= \text{Max } [N(X_{T-2}) - N(S_{T-2}) + \rho E_{T-2}\{V_{T-1}\}] \\ &= \text{Max } [\rho E_{T-2}\{N(z_{T-1} G(S_{T-2}))\} - N(S_{T-2}) + N(X_{T-2}) - \rho E_{T-2}\{N(S^*_{T-1})\} \\ &\qquad + \rho^2 E_{T-2}\{N(z_T G(S^*_{T-1}))\} - \rho^2 N(c/(pq))] \end{aligned} \tag{89}$$

In period t=T-2, X_{T-2}, S^*_{T-1}, and $S^*_T = c/(pq)$ are given, and with the z_{t+1} being i.i.d., the last four terms in (89) are constants. The optimal level of escapement, S^*_{T-2} , that maximizes (89) also maximizes

$$W(S_{T-2}) = \rho E_{T-2}\{N(z_{T-1} G(S_{T-2}))\} - N(S_{T-2}) \tag{90}$$

Compare equations (90) and (85). While the period index is one period earlier in equation (90), equations (85) and (90) have the same form and if we continued to work our way back in time the same form would emerge for $W(S_{T-3})$, $W(S_{T-4})$, and so on, back to $W(S_0)$.

Now let $T \to \infty$. In any period the optimal escapement policy, S^*, must maximize

$$W(S) = \rho E_z\{N(zG(S))\} - N(S) \tag{91}$$

Taking a derivative, and noting again that $G'(S)$ is independent of z, S^* must also satisfy

$$G'(S)\left[\frac{E_z\{zN'(zG(S))\}}{N'(S)}\right] = (1+\delta) \tag{92}$$

Numerically it is perhaps easier to do a computer search for the S that maximizes (91) than the S which satisfies (92), but they must be the same. Once we find the optimal (stationary) level of escapement, the optimal (adaptive) harvest policy takes the form

$$Y_t^* = (X_t - S^*) \text{ if } X_t > S^*$$
$$Y_t^* = 0 \text{ if } X_t \le S^* \tag{93}$$

Stock, harvest, and effort will be fluctuating over time. Given a probability density for z, say f(z), it is unlikely that one would be able to determine a closed-form distribution, $\omega(X|S^*)$, but one could simulate the optimal harvest policy over a long horizon (say, T=1,000) and calculate the mean, variance, or other descriptive statistics for X_t, after a suitable transition from X_0.

As a numerical example, consider the case when $X_{t+1} = z_{t+1} S_t e^{r(1-S_t/K)}$, where r > 0 is an intrinsic growth rate for a race of Pacific salmon, K > 0 is an environmental carrying capacity (number of salmon) and the z_{t+1} are i.i.d. as given by

$$\Pr(z_{t+1} = z_1 = 0.5) = 0.25$$
$$\Pr(z_{t+1} = z_2 = 1.0) = 0.50 \tag{94}$$
$$\Pr(z_{t+1} = z_3 = 1.5) = 0.25$$

We are assuming that $E\{X_{t+1}\} = G(S_t) = S_t e^{r(1-S_t/K)}$, but that X_{t+1} (number of salmon) could be 50% less (z_1 = 0.5) or 150% more (z_3 = 1.5), each with probability 0.25. Recall that N(m) = pm - (c/q)ln(m). Suppose r=2, K=200000, p=2, q=1, c=4000, and δ=0.05. Then the value of S which maximizes W(S), as given by equation (91), and which satisfies equation (92), is S^* = 71,534 (salmon).

3.6 The Stochastic Fishery: Continuous Time

Consider a fishery where the stock evolves according to

$$dX = [rX(1 - X/K) - Y]dt + \sigma X dz \tag{95}$$

Note that the expected response to harvesting is dX/dt = rX(1 - X/K) - Y, but the standard deviation rate, σ > 0, results in a variation about the expected change, and that variation increases with the size of the stock.

The net benefit at instant t is given by

$$\pi(X,Y) = \int_0^Y (b/\omega)^2 d\omega - cY/X^2 = -b^2/Y - cY/X^2 = -\frac{(b^2X^2 + cY^2)}{YX^2} \tag{96}$$

which is a strange-looking expression, but results from two assumptions (i) that the benefit from harvest at rate Y is the area under an iso-elastic inverse demand curve where $p = (b/Y)^2$, b > 0, and (ii) the cost of harvesting Y form a stock of size X is given by cY/X^2, c >0. To take the integral one must transform variables, defining Y=b/sinθ. Given the forms in equations (95) and (96) the H-J-B equation, which was given in equation (47) in Subsection 2.4, becomes

$$\delta V(X) = \text{Max}\left[-\frac{(b^2X^2 + cY^2)}{YX^2} + [rX(1 - X/K) - Y]V'(X) + (\sigma^2/2)X^2V''(X)\right] \quad (97)$$

The maximal condition requires $\partial[\cdot]/\partial Y = 0$ and implies

$$(b/Y)^2 - c/X^2 = V'(X) \quad (98)$$

Note again that $V'(X)$ in the stochastic model is playing the same role as μ from the continuous-time deterministic model. Basically $(b/Y)^2 - c/X^2$ = price - marginal cost = rent, so $V'(X)$, as μ earlier, can be regarded as the rent or net revenue on the last fish harvested at instant t. We can solve (98) for Y as a function of X and $V'(X)$ yielding

$$Y^* = \frac{b}{(V'(X) + c/X^2)^{1/2}} \quad (99)$$

Substituting the expression for Y^* into equation (97) we will have accomplished the maximization and we can write, after some careful algebra,

$$\delta V(X) = -2b(V'(X) + c/X^2)^{1/2} + rX(1 - X/K)V'(X) + (\sigma^2/2)X^2V''(X) \quad (100)$$

This is a second-order differential equation which somewhat amazingly has a closed-form solution

$$V(X) = -\varphi/X - \varphi r/(\delta K) \quad (101)$$

where $V'(X) = \varphi/X^2$, $V''(X) = -2\varphi/X^3$, and

$$\varphi = \frac{2b^2 + 2b\sqrt{b^2 + (r + \delta - \sigma^2)^2 c}}{(r + \delta - \sigma^2)^2} > 0 \quad (102)$$

[To verify the expression for φ, and to convince yourself that (101) is a solution to (100), you need to substitute $V(X)$, $V'(X)$, and $V''(X)$ into (100) and show that it holds provided φ is as given in (102).]

With $V'(X) = \varphi/X^2$, we can go back to equation (99) and write our optimal (adaptive) policy as

$$Y^* = \frac{b}{[\varphi/X^2 + c/X^2]^{1/2}} = b(\varphi + c)^{-1/2}X \quad (103)$$

Equation (103) reveals that the optimal harvest policy is *linear* in X. In X-Y space it is a ray through the origin with slope $b(\varphi + c)^{-1/2} > 0$.

It can be shown that $\partial\varphi/\partial\sigma^2 > 0$, and that stochastic fluctuations reduce V(X). Fluctuations increase $V'(X)$ (rent) and will reduce optimal harvest Y^* at every stock level, X.

Substituting Y^*, as given in (103) back into (95) results in

$$dX = [rX(1 - X/K) - b(\varphi + c)^{-1/2}X]dt + \sigma X dz \tag{104}$$

A sufficient condition for a non-degenerative distribution of X is $\sigma^2 < 2r - 2b(\varphi + c)^{-1/2}$, in which case the distribution for X is given by

$$\omega_\infty(X) = \frac{(2r/(\sigma K))^{(2\theta/\sigma^2 - 1)} X^{(2\theta/\sigma^2 - 2)} e^{-2rX/(\sigma^2 K)}}{\Gamma(2\theta/\sigma^2 - 1)} \tag{105}$$

where $\theta = r - b(\varphi + c)^{-1/2}$ and $\Gamma(\cdot)$ is the gamma function. After a transition to the stationary distribution, $\omega_8(X)$, the expected stock size and harvest are given by

$$\bar{X}_\infty = K\{1 - \sigma^2/(2r) - b/[(r(\varphi + c)^{1/2}]\} \tag{106}$$

and

$$\bar{Y}_\infty = K\{[1 - \sigma^2/(2r)]b/[(\varphi + c)^{1/2}] - b^2/[r(\varphi + c)]\} \tag{107}$$

As a numerical example, suppose b=10, c=4,000, r=2, K=200,000, δ=0.05, and σ^2=0.04. The non-degeneracy condition is satisfied, φ=680.76, $\bar{X}_\infty$=183,384 and $\bar{Y}_\infty$=26,804. We might approximate the evolution of X_t and Y_t^* by the system

$$\begin{aligned} X_{t+1} &= X_t + rX_t(1 - X_t/K) - Y_t^* + \sigma X_t z_t \\ Y_t^* &= b(\varphi + c)^{-1/2} X_t \end{aligned} \tag{D2}$$

where $z_t \sim N(0,1)$. Figure 7 shows some sample paths for X_t and Y_t^* starting from $X_0=\bar{X}_\infty$ and $Y_0=\bar{Y}_\infty$ for T=100. Because of the linear harvest policy, the time path for Y_t^* moves up and down with X_t. One unusual feature of this model is that as $b \to \infty$, $b(\varphi + c)^{-1/2} \to 1$, and $Y_t^* \to X_t$.

(Figure 7. about here)

4. Impediments to Resource Management

Why have so many ecologists and economists put so much time and effort into developing models of renewable resources? A short answer is that most societies, through time and across cultures, have not done particularly well at managing such resources. More detailed explanations for the failure of a society to effectively allocate renewable resources are based on theories of common property under open access, poverty, information, and collective institutions. Let's examine some of these impediments to resource management and, in light of some of the models from the previous sections, try to identify policies that might improve their allocation over time.

4.1 Common Property, Open Access, and Missing Shadow Prices

We might define a common property resource as one that is exploited by a group of individuals. The size of the group and the conditions imposed on its members might range from a small isolated tribe, where certain individuals have the right to harvest wild honey, to an international fleet of modern vessels, harvesting tuna in the South Pacific. If access to the resource is unrestricted and harvest unregulated, the situation is referred to as *pure open access*, or *res nullius*. A distinction has been made between pure open access and *regulated open access*, where the access to a fish stock may be open (unrestricted), but the overall level of harvest is restricted by an annual quota imposed by a government. Depending on the size of the group, community norms or regulations, and the degree of enforcement, different incentives for resource use are created.

It has long been maintained that open access to a common property resources would result in the *dissipation of rent* (zero net revenue) and over-exploitation of the resource. Open access dynamics might be described by the system

$$\begin{aligned} \dot{X} &= rX(1 - X/K) - qXE \\ \dot{E} &= \eta[pqXE - cE] \end{aligned} \tag{D3}$$

where we have assumed logistic growth, $F(X) = rX(1 - X/K)$, and a production function where $Y = qXE$ (see the Linear Model of Subsection 3.1). The change in effort, $\dot{E}$, is assumed to be proportional to net revenue, $\pi = pqXE - cE$, where $\eta > 0$ is a stiffness or adjustment parameter. If net revenue is positive, effort increases. If net revenue is negative, effort decreases. Open access equilibrium occurs at $X_\infty = c/(pq)$, from $\dot{E} = 0$, and $E_\infty = (r/q)(1 - X_\infty/K)$, from $\dot{X} = 0$, with net revenue being driven to zero in equilibrium. The open access equilibrium can be shown to be a stable node or the focus of a stable spiral. A model similar to (D3) has been used to explain the dynamics of the human population and resource base on Easter Island.

It is usually the case that X_∞ is considerably smaller than the bioeconomic optimum, X^*, and that effort E_∞ is considerably greater than E^*. This has led to the description of open access equilibrium as a situation where "too many vessels are chasing too few fish." This "tragedy of the commons," while likely, need not always occur. It is possible to show that cooperative behavior in a small, closely-knit, community may lead to a stable equilibrium without over exploitation. Such communities, however, are susceptible to shocks from outside agents, or changes in technology, that can destroy the cooperative equilibrium and cause a structural change in the dynamics to a system similar to (D3), where the stock evolves to X_∞.

Missing in models of pure open access is any incentive to leave units of the resource *in situ* if they could be harvested for positive net revenue today. A common property resource, harvested under open access, is treated as if it had a zero price.

In problems (P1) and (P2), the maximal conditions implied that $\partial p(\cdot)/\partial Y = \mu$ [from equation (15)] or $\partial \pi(\cdot)/\partial Y_t = \rho\lambda_{t+1}$ [from equation (25)]. In each problem, optimal harvest required that the net marginal benefit from harvesting an additional unit of the resource today ($\partial \pi(\cdot)/\partial Y$ or $\partial \pi(\cdot)/\partial Y_t$) be equated to the shadow price of the resource [μ in Problem (P1) or $\rho\lambda_{t+1}$ in Problem (P2)]. These shadow prices are absent, or inoperable, in pure open access, and without them, the resource is typically harvested to very low levels. Economists have proposed landings taxes or individual transferable quotas (ITQs) as a way of introducing opportunity costs to harvest which might approximate the shadow prices in Problems (P1), (P2), and elsewhere in this essay. (We have already noted that $V'(X)$ in equation (48) is playing a similar role to μ and $\rho\lambda_{t+1}$.)

Landings taxes have been strongly opposed by fishing groups and, in fact, were explicitly forbidden when the Magnuson Fishery Management and Conservation Act was initially enacted in the U.S. in 1976. Fishers were more open to using individual transferable quota (ITQ), where an individual fisher was granted the right to harvest a fraction (share) of an annual quota set by management authorities. The holder of an ITQ could decide when and where to fish (within certain limits), or whether to sell or rent the quota to another fisher. One of the impediments to the use of individual transferable quotas is that they typically require *limited access* to the resource. This basically results in granting a property right to a small group of fishers who have the potential to make significant profits. There are ways to "tax back" profits, if they are deemed excessive, and to periodically re-select the fishers granted access to the fishery, say, via a lottery. ITQs are now being used in New Zealand, Australia, Iceland, Canada, and to a limited extent in the U.S.

4.2 Poverty

Perhaps the greatest impediment to resource management in the developing world is poverty. People on the edge of starvation are not in a position to reduce harvest today in order to invest in the stock tomorrow. The depletion of groundwater, grassland, and forest resources have been widely documented in the Third World, where human population growth, limited capital and technology, and sometimes civil war, force people to harvest any resource that can be consumed or sold. Under such circumstances it is actually rational to have a very high rate of time preference or discount. In the models of fishery, forest, and groundwater resources considered in this essay, an increase in the discount rate, δ, will lead to a decrease in the optimal fish stock, X^*, to a shortening of the rotation for an even-aged forest, and to a lower stock of groundwater.

In the linear fishery model of Subsection 3.1, as $\delta \rightarrow \infty$, $X^* \rightarrow X_\infty$. In words, as the discount rate becomes infinitely large it is optimal to harvest a resource down to its open access level, where the resource is treated as though it had a zero price. It would seem entirely plausible that people faced with starvation and civil war would have nearly infinite rates of discount. For them, there may literally be no tomorrow. So it should not be surprising that we see impoverished people harvesting endangered wildlife in Africa, clearing rain forest in Indonesia, or fishing with dynamite in the Philippines. It is perhaps more amazing that wildlife, rain forests, and coral reefs still exist.

It is easy to become depressed about the prospects for resource management and conservation in the Third World. There are, however, a few policies that have provided some relief for impoverished people and natural environments under stress. One policy attempts to endow local communities with a property right or a "financial stake" in a park, preserve, or particular species. This has been accomplished though (i) eco-tourism, (ii) the sale of species-specific hunting licenses to affluent foreigners, or (iii) payments and royalties from a pharmaceutical firm to a federal government or local cooperative for the exclusive rights to bio-prospecting within a tropical ecosystem. In all three cases, local people, who might otherwise harvest resources or clear the forest, are given jobs in the tourist industry or share in the revenues from sport hunting or the collection of biologically active chemicals that might lead to new, disease-fighting drugs. These people would presumably have an incentive to protect and maintain a natural environment or the "desirable" species within such an environment. Game hunting in Zimbabwe and eco-tourism and bio-prospecting in Costa Rica are oft-cited examples.

Providing substitutes for natural resources is another strategy. In Nepal, villagers were provided with gasoline cook stoves in an attempt to reduce the demand for fuel wood. Pastoralists might be encouraged to raise goats instead of cattle, if they are more efficient at converting grass to milk and meat. Sometimes the substitute might be an indigenous species. In Africa, there have been attempts at game ranching, since wildebeest and gazelle are less damaging to grassland and require less water than exotic cattle.

In theory, carefully targeted foreign aid might help people grow food and reduce pressure on forests, wildlife and the remaining natural environments. The effective use of foreign aid has proven difficult, and frequently projects of good intent fail because the technology is inappropriate, the motives of farmers are not clearly understood, or corrupt government officials divert project resources and limit the amount of aid actually reaching project participants.

4.3 Information

The models of the previous sections have two common components, an objective function and a net growth function. Both require time-series data to estimate the parameters of alternative functional forms. In developed countries, there are often agencies charged with the responsibility for collecting data on harvest and market prices, and estimating the size of a resource stock over time. For fisheries, stock size is often estimated through statistical models which pool data on commercial harvest with data from "scientific fishing," and which allow the estimate of biomass or the size of particular cohorts to be updated each year. Forest resources are somewhat easier to inventory, although the actual volume of merchantable timber from a parcel is never known with certainty until it has been milled. The size of an aquifer and the volume of water in saturated sediments is also easier to estimate than a fish stock, but may require fairly expensive sample wells to be drilled in order to assemble the necessary hydrological information on storativity, recharge, and return flow. Establishing agencies, employing the appropriate research scientists and administrative staff, and collecting the right time-series data (in a consistent way), is expensive. It is a luxury that developing countries may not be able to afford. Even in developed countries, there are data relating to economic net value that are not measured on an annual or regular basis. For example, data on effort and the cost of harvest may not be collected annually, yet they are needed if economists are to estimate net revenue or consumer and producer surplus.

Most bioeconomic models are concerned with the optimal management of a single species. In modeling a marine fishery there may be other species which predate, compete, or serve as prey for the species of interest. A *multi-species system* may be needed to understand population dynamics and the net growth of a commercially harvested species. This can create significant information costs. It is often expensive to fund research projects which would estimate coefficients of predation, if one species preys on two or more species, or coefficients of competition, if two or more species compete for the same prey (food) species. Multi-species models will often suggest management strategies that would not emerge in a single-species model. For example, in the Southern Ocean, surrounding Antarctica, the different species of baleen whale (blue, fin, sei, and minke), as well as penguins and other sea birds, all feed on krill (a shrimp-like crustacean). Between WWI and WWII, and immediately following WWII, commercial whalers took large numbers of blue, fin and sei whales, leaving a surplus of krill for the minke whale, penguins, and seabirds. Minke whales in the Southern Ocean are probably in greater abundance now than immediately after WWII, and their high numbers may actually slow the recovery of blue whales, which were threatened with extinction. Allowing the Japanese to harvest a greater number of minke whales might actually hasten the recovery of blue and fin whales, by reducing the competition for krill.

The costs of collecting data on harvest and effort, as well as biological and economic research, can sometimes be shared by countries harvesting the same resource. For example, the Inter-American Tropical Tuna Commission collects data for member nations on harvest rates, vessel days, and prices, and provides scientific estimates of stock size and the value and distribution of catch. Tuna is a commercially-important, migratory species, and a cooperative, impartial research organization is important in allocating quota among member countries. A similar organization exists for tuna in the tropical Atlantic.

4.4 Incomplete Institutions

In addition to institutions with a mandate to monitor, study and manage renewable resources, it is important to coordinate policies between different levels of government within a nation state and, where necessary, coordinate management between nation states. Credible enforcement of management regulations is needed nationally and internationally. The need for intergovernmental and international institutions would seem to vary from forest to water to fishery resources. Lack of coordination or weak enforcement of management regulations might lead to *de facto* open access.

Forested lands might be owned by individuals, firms, or local, state, or federal governments. Individuals might own forested land for a variety of reasons, including the harvest of fuel wood and timber, or the amenity value of providing habitat for wildlife. A large, forest-product company will typically own an inventory of forested lands, in order to provide timber or fiber to their mills. Governments might own forested land as part of a system of parks or as part of an inventory which is managed for timber, wildlife habitat, and recreation (*ie*, multi-purpose management). Governments owning large tracts of forest land, or countries with large forest-product industries, will often have agencies with management, research, or educational responsibilities. These agencies may provide advice on disease control, future market conditions, or help small woodlot owners with management. While there may be efforts to coordinate research and education between state and federal agencies, with the possible exception of disease, there may not be a compelling need for international coordination in forestry.

Groundwater, or more generally water resources, are valuable and have traditionally been managed through municipal, regional, state, and federal agencies. These agencies are often dealing with issues of water supply and distribution, as well as water quality. In arid and semi-arid environments, access to water can be critical, and has even lead to armed conflict. Governments have built huge reservoirs and canal systems to provide water for cities and agriculture. There is probably a more compelling need to coordinate water management between levels of government within a country and between countries that border on, or jointly exploit, surface or groundwater resources. The U.S. has treaties with both Canada and Mexico dealing with water resources. One of the many obstacles to peace in the Middle East is the level of use and rights of access to water.

We have already noted the role of institutions in collecting data, monitoring fishing effort and catch, enforcing regulations, and performing biological and economic research. Such information is critical to management in general, and to the construction and estimation of bioeconomic models in particular. Most coastal countries will typically have an agency responsible for fishery management within their territorial waters. For migratory species, like tuna, whales, or herring, management must be coordinated between countries to avoid sub-optimal outcomes. In addition to the previously mentioned commissions for tuna, there are international commissions for whales, halibut, and marine mammals in the North Atlantic. These international commissions have the difficult task of trying to cooperatively restrict aggregate landings so as maintain or re-build stocks harvested by several nation states. Each country may have the capacity to overfish the stock, and thus, each possesses a credible threat point. Trans-boundary, or fish stocks which "straddle" the territorial waters of two or more nations present a compelling need for international institutions that can objectively (and accurately) estimate stocks, propose equitable quota for member countries, and monitor annual catch.

See: **Economics of Environmental Regulation**, **Environmental Economics**, **Strategic Behavior**, **Sustainable Growth**, **Macroeconomics**, **Economic Externalities**, **Development Economics**, **Public Goods**

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Author Biography

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Figure 1. F(X)=rX(X/k-1)(1-X/K), r=1, k=0.25, K=1

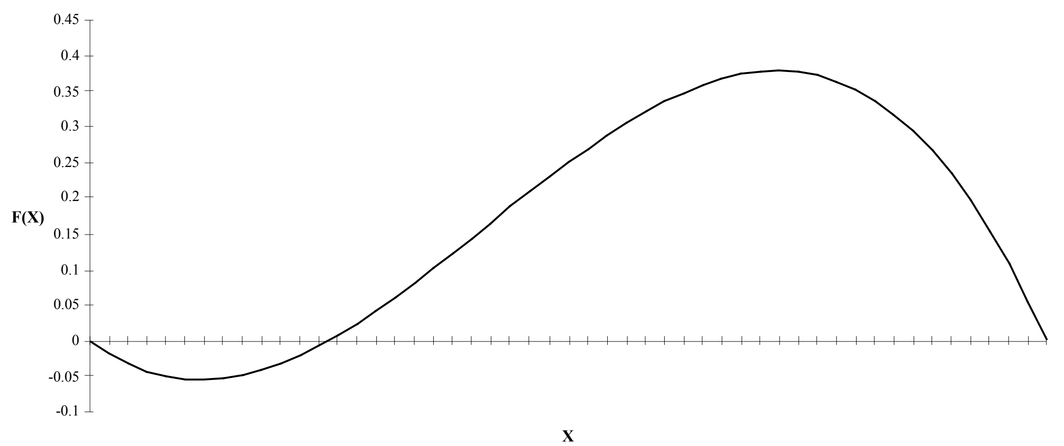


Figure 2. The X-Y Phase Plane when π=ln[Y]+ßln[X] and F(X)=rX(1-X/K), r=1, K=1, Beta=4, Delta=0.02

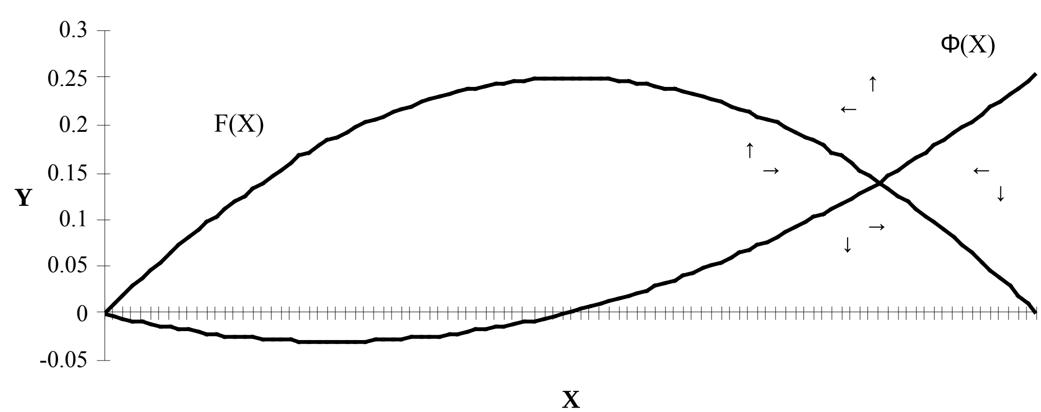


Figure 3. The Approximately Optimal Approach from X(0)=0.2 to X*=0.83 when π=ln[Y]+ßln[X], F(X)=rX(1-X/K), r=1, K=1, Beta=4, Delta=0.02

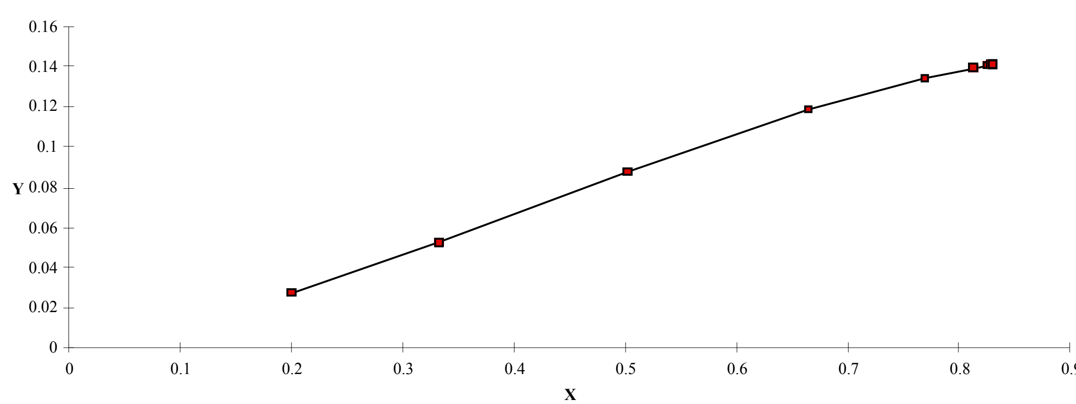


Figure 4. The MRAP Approach to X* from X∞=c/p

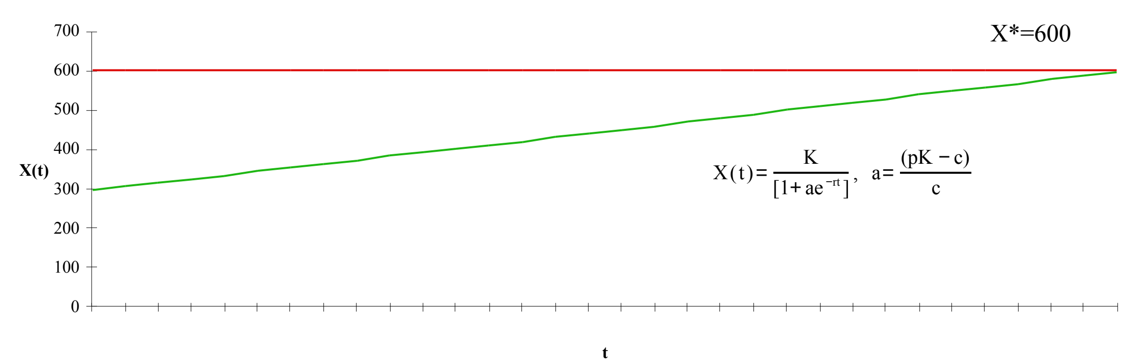


Figure 5. The Volume Function Q(t)=EXP(a-b/t) when a=13 and b=196

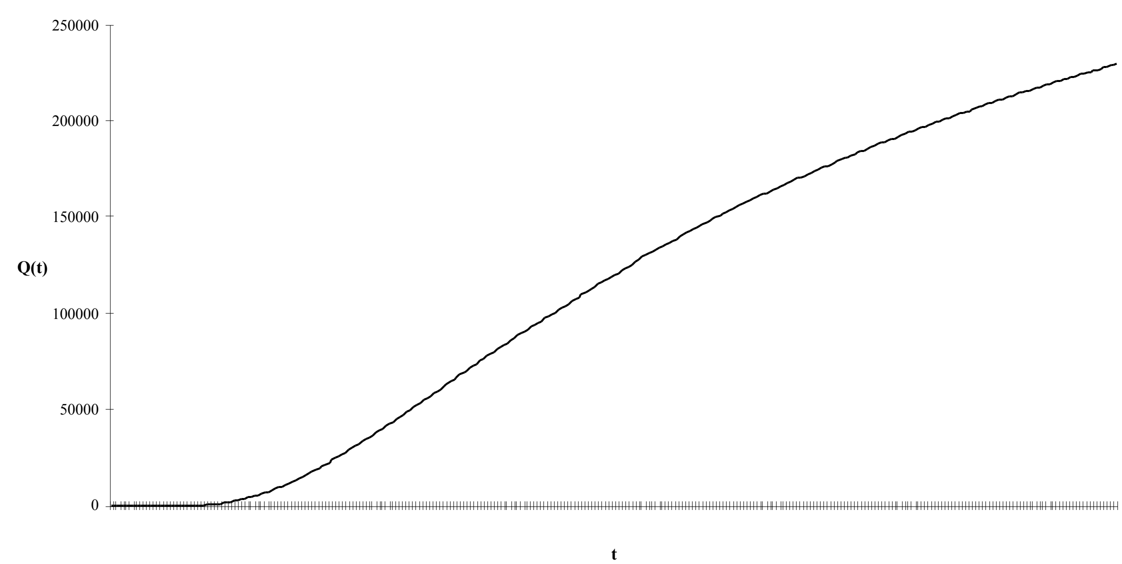


Figure 6. The Optimal Groundwater Stock

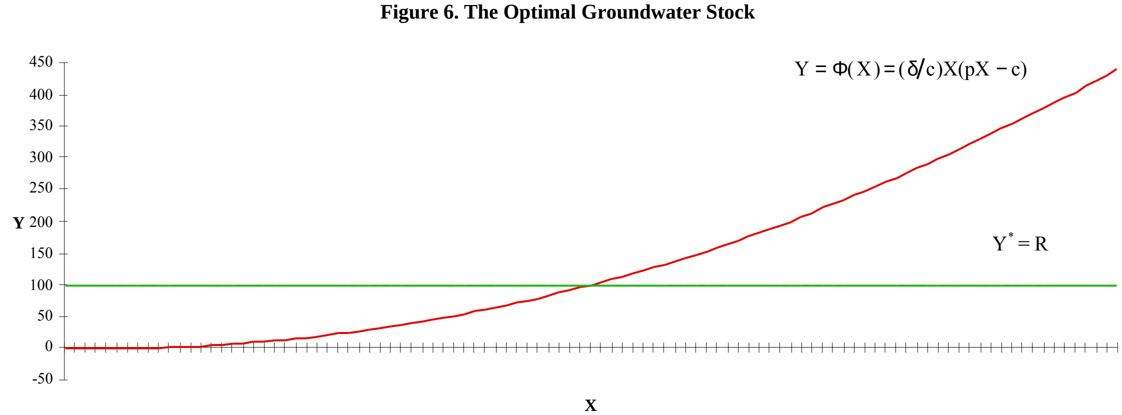
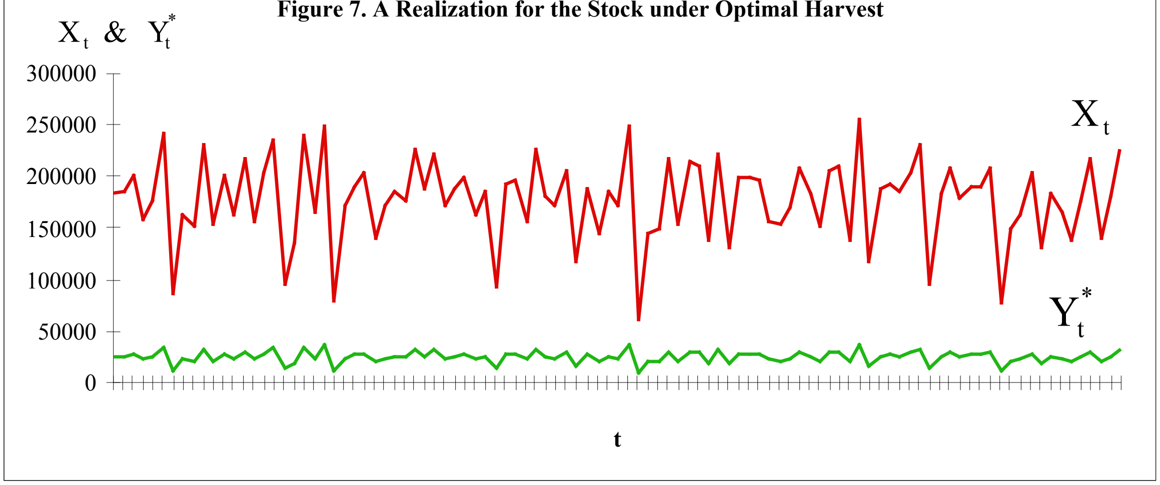


Figure 7. A Realization for the Stock under Optimal Harvest



Approximately Optimal Approach to the Steady State Optimum
when $\pi = \ln[Y] + ß\ln[X]$ and $F(X) = rX(1-X/K)$

r=	1
K=	1
ß=	4
δ=	0.02

t	Xt	Yt	Discounted πt
0	0.2	0.1	-8.740336743
1	0.26	0.1	-7.540078122
2	0.3524	0.1	-6.223124407
3	0.48061424	0.1	-4.931499196
4	0.630238432	0.1	-3.833231786
5	0.763276383	0.1	-3.064200925
6	0.843961929	0.1	-2.647199564
7	0.87565212	0.1	-2.466933062
8	0.884537605	0.1	-2.384093997
9	0.886668435	0.1	-2.329293856
10	0.887155956	0.1	-2.281817699
11	0.887266222	0.1	-2.23667635
12	0.887291095	0.1	-2.192731535
13	0.887296703	0.1	-2.149717257
14	0.887297967	0.1	-2.10756162
15	0.887298252	0.1	-2.066235928
16	0.887298316	0.1	-2.025721287
17	0.88729833	0.1	-1.986001215
18	0.887298334	0.1	-1.947060005
19	0.887298334	0.1	-1.908882355
20	0.887298335		-95.44411772
		PVNB=	-160.5065146

Approximately Optimal Approach to the Steady State Optimum
when π=ln[Y]+ßln[X] and F(X)=rX(1-X/K)

r=	1
K=	1
β=	4
δ=	0.02

t	Xt	Yt	Discounted πt
0	0.2	0.027288091	-10.0390565
1	0.332711909	0.052622292	-7.20247907
2	0.502104311	0.08721279	-4.99345813
3	0.664887092	0.118206555	-3.55055171
4	0.769492784	0.134061888	-2.82470224
5	0.812804536	0.13925569	-2.53650097
6	0.825702168	0.140610505	-2.42224932
7	0.829009761	0.140938896	-2.35880217
8	0.829823443	0.14101912	-2.30871627
9	0.830020819	0.141044417	-2.26250123
10	0.830062661	0.141058848	-2.21788912
11	0.830062453	0.141073004	-2.17432119
12	0.830048226	0.141088437	-2.13165525
13	0.830027957	0.141105755	-2.08983871
14	0.830003749	0.141127483	-2.04883321
15	0.829973792	0.141158459	-2.00860422
16	0.82993263	0.14120132	-1.96914318
17	0.829875769	0.1412046	-1.93071166
18	0.829853146	0.140898179	-1.89445195
19	0.830151869	0.141145517	-1.8551137
20	0.830006095		-92.7918429
		PVNB=	-153.611423